FDL-TDR-64-152
PART VII

UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

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L. V. ANDREW and T. E. STENTON
North American Rockwell Corporation

TECHNICAL DOCUMENTARY REPORT No. FDL-TDR-64-152, PART VII

AUGUST 1968

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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FOREWORD

This report covers a portion of the research conducted by the Los Angeles Division of North American Rockwell Corporation, Los Angeles, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF33(615)-2896.

The work was performed to advance the state-of-the-art of flutter prediction for flight vehicles as part of the Air Force Systems Command exploratory development program. The research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles", Task No. 137003 "Prediction and Prevention of Aero-thermoelastic Problems". Messrs. James J. Olsen and Samuel J. Pollock of the Aerospace Dynamics Branch were Project Engineers.

Mr. H. Hoge was the Program Manager for North American Rockwell. Mr. L. V. Andrew and Mr. T. E. Stenton were Principal Investigators. The basic approach was outlined by Dr. M. T. Landahl of the Massachusetts Institute of Technology. The calculus of variations approach was suggested by Mr. James Olsen.

The contractor's designation of this report is NA-66-694.

Manuscript released by the Authors in October 1967 for publication as an AFFDL Technical Report.

This technical report has been reviewed and is approved.

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ABSTRACT

Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.

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SYMBOLS

c	chord
C	speed of sound
g	time of travel of an acoustic signal
M	Mach number
r	Slope in the y-direction, dx/dy
δR	Increment in radius vector
s	Distance along a ray path, span
t	Time
U	Free-stream velocity
V	Velocity
x, y, z	Location of a field point
x_o, y_o, z_o	Location of a source or doublet point
X, Y	$x/\beta s$, y/s
X^*, Y^*	Linear transformation of coordinates X, Y
$\hat{i}', \hat{j}', \hat{k}'$	Unit vectors along x', y', z' axes
$\mathbb{R}$	Radius vector
∇	Vector gradient operator, $\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
β	$\sqrt{1-M^2}$, $\sqrt{r^2 + 1-M^2}$
δ	$\sqrt{5 + M^2}$, an increment
ϕ	Velocity potential
Λ	Ray Angle
τ	Thickness ratio

Subscripts

a	Advancing
ℓ	Local, lower

r	Receding
u	Upper
x, y	Partial derivatives with respect to x, y
c	Sonic line
∞	Infinity

Superscripts

$\cdot$	Derivative with respect to time
'	Derivative with respect to the independent variable

INTRODUCTION

When an airfoil travels through the air at speeds near the speed of sound, the local speed of flow varies from subsonic near the forward edges to supersonic near the trailing edges. These wide variations of speed from that of the free-stream characterize the non-uniform transonic flow. This non-uniformity of the flow field must be accounted for in accurate calculations of unsteady pressures and forces; particularly their phase lags.

In order to determine an unsteady transonic flow field one requires solutions for singularities immersed in a non-uniform steady flow, (Reference 1). Source solutions for a mean flow that varied in the x-direction only were given in the high-frequency limit by Landahl (Reference 2). Rodemich (Reference 3) presented a "box" solution, based on pulsating doublets, which assumes a uniform mean flow at Mach number 1.0. No exact solutions for the case of a mean flow with arbitrary spatial variations have been found, thus far, but Landahl proposed the basic form of a solution which removes most of the limitations and restrictions of these approximate solutions. The method focuses attention on the time of transmission of an acoustic signal from a pulsating sending source to a distant receiving point. The signal travels through a nearly sonic flow field where the Mach number varies in a prescribed manner.

This report contains a difference equation approach, and differential equation approach to computing the paths and the transmission times for acoustic signals. The independent variable in the latter approach is a spatial rather than a time variable. A procedure that could be used to calculate the velocity potentials and generalized forces on an oscillating surface is described.

POTENTIAL OF A UNIT SOURCE

The basic expressions proposed by Landahl for the velocity potential at the point (x,y,z) due to a pulsating source at $(x_o,y_o z_o)$ are:

(a) for a source in a locally subsonic flow region

$$\phi = \frac{-1}{4\pi\bar{R}} \exp \{i\omega[t-g(x,y,z,x_o,y_o,z_o)]\} \tag{1}$$

where

$$\bar{R} = \sqrt{(x-x_o)^2 + [1-M^2(x,y,z)][(y-y_o)^2 + (z-z_o)^2]}$$

M = Local Mach Number

x_o,y_o,z_o = Location of source point

$g(x,y,z,x_o,y_o,z_o)$ = Time required for a disturbance to travel from (x_o,y_o,z_o) to (x,y,z).

(b) for a source in a locally supersonic flow region

$$\phi = \frac{-1}{4\pi\bar{R}} \{\exp[i\omega(t-g_a)] + \exp[i\omega(t-g_r)]\} \tag{2}$$

where

$g_{a,r} = g_{a,r}(x,y,z,x_o,y_o,z_o)$ = Time required for the advancing, receding wave to travel from (x_o,y_o,z_o) to (x,y,z)

It is likely that good accuracy may be obtained with use of the value of g_a for uniform flow (in the supersonic case, and also for the advancing wave portion in the subsonic case). However, our purpose is to produce a general solution for g which applies to both the advancing and the receding portions of the wave and compare values with those for uniform flow.

Since the primary interest is in wing flows, we consider that both the source and receiver points lie in the x, y-plane, so that $z = z_o = 0$. Furthermore, we consider that signals do not return to the plane once they leave. The problem is thus simplified to one in two spatial dimensions. Its solution should be applicable to a wide variety of nearly planar lifting surfaces.

Consider a signal emanating from a source at the point (x_o,y_o) on a wing. A second point past which the signal travels is located an incremental distance (dx, dy) away. There are two components of velocity of the signal, a radial component, C, where C is the local speed of sound and an x-component, U, where U is the local speed of flow over the wing. Λ is the angle the radial component makes with the negative extension of the x-axis. The path of this wavefront point will be referred to as a "ray". The shape of any ray depends on the initial choice of Λ; for a given Λ, dx and dy are components of the first element of this particular ray emanating from (x_o,y_o). The situation depicted is general in that it applies not only at the source, but at any point on the ray path. Thus, the

velocity at any point on the path is a function of three spatial parameters which vary with position, U, C, and Λ. From the sketch, it is clear that

$$dx = [U(x, y) - C(x, y) \cos \Lambda]\, dt \qquad (3)$$

$$dy = C(x, y)\, dt \sin \Lambda$$

Equations were developed for two methods of tracing the ray path to establish the magnitude and the phase relationship at field points to a unit source. These methods are: (1) a difference equation method, and (2) a non-linear differential equation method.

Difference Equation Method

In this method, time is the independent variable. Equations (3) are two of the three equations needed to establish the variation of x, y, and Λ with time. The third equation is obtained by considering the acceleration of the ray in the non-uniform flow field (see Figure 1).

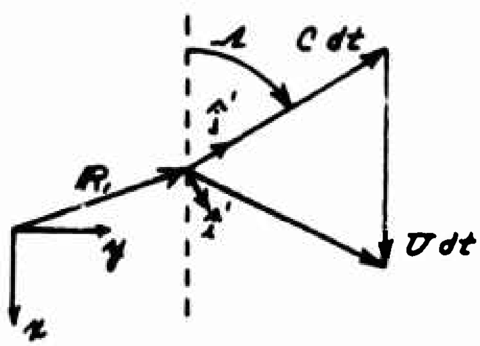


Figure 1. Velocity Components of a Sonic Ray Line In A Moving Airstream

In terms of components in the directions of the rotating unit vectors $\hat{i}'$ and $\hat{j}'$

$$\dot{R}_1 = (U \sin \Lambda)\,\hat{i}' + (C - U \cos \Lambda)\,\hat{j}' \qquad (4)$$

and

$$\ddot{R}_1 = (\dot{U} \sin \Lambda + C\dot{\Lambda})\,\hat{i}' + (\dot{C} - \dot{U} \cos \Lambda)\,\hat{j}'$$

It is necessary to express the angular velocity $\dot{\Lambda}$ in terms of space variables. To do this, consider that at time t a second ray point is located at $R_2 = R_1 + \delta R\,\hat{i}'$, where δR is small, and it's direction of travel is $\dot{R}_2 = \dot{R}_1 + \delta\dot{R}$. Let the superscripts (o) and (1) denote times t_o and $t_1 (= t_o + \Delta t)$. Then at time t_1

$$R_1^{(1)} = R_1^{(0)} + \dot{R}_1^{(0)} \Delta t$$

and $$R_2^{(1)} = R_2^{(0)} + \dot{R}_2^{(0)} \Delta t$$

Subtracting the first equation from the second

$$\delta R^{(1)} = \delta R^{(0)} + \delta \dot{R}^{(0)} \Delta t \tag{5}$$

where $$\delta R = R_2 - R_1$$

Recalling that the cross product of two vectors is a vector normal to the plane defined by the two vectors, and has a magnitude equal to the product of the two magnitudes times the sine of the angle between them, then

$$\delta R^{(0)} \times \delta R^{(1)} = \hat{\lambda}' \left(-\delta R^{(0)} \delta R^{(1)} \sin \Delta \Lambda \right) \tag{6}$$

which has the correct sense. When $\Delta\Lambda$ is small, and when Equation (5) is substituted into the left side of Equation (6), we get

$$\delta R^{(0)} \times \delta \dot{R}^{(0)} \Delta t = \hat{\lambda}' \left(-\delta R^{(0)} \delta R^{(1)} \Delta \Lambda \right)$$

This may be rewritten as

$$\frac{\Delta \Lambda}{\Delta t} = - \frac{\delta (C - U \cos \Lambda)}{\delta R^{(1)}}$$

and in the limit as $\Delta t \to 0$

$$\dot{\Lambda} = -\hat{\lambda}' \cdot \nabla (C - U \cos \Lambda) \tag{7}$$

where the operator $\hat{\lambda}' \cdot \nabla$ is

$$\hat{\lambda}' \cdot \nabla = \left(\sin \Lambda \frac{\partial}{\partial x} + \cos \Lambda \frac{\partial}{\partial y} \right)$$

and operates only on C and U.

Equation (7) has a revealing physical interpretation. From Figure 4 we see that the gradient of the speed of sound C, on forward portions of the wing, is a vector pointing forward and slightly outward from the center-line; whereas, from Figure 3 we see that the gradient of the local flow speed U is nearly in the opposite direction. Although it is not apparent from the figures because they are plotted to different scales, the magnitude of the gradient of U is about five times that of the gradient of C. From the energy equation $C^2 + \frac{\gamma - 1}{2} U^2 = \text{constant}$, $\nabla U = -5.0 \, \nabla C$. The local Mach number is increasing in the downstream direction. Figure 2 shows that, under these conditions there are only two stable ray angles; those for which the gradient of $C - U \cos \Lambda$ is zero. As the ray propagates through the flow field it will always tend towards one of these two orientations.

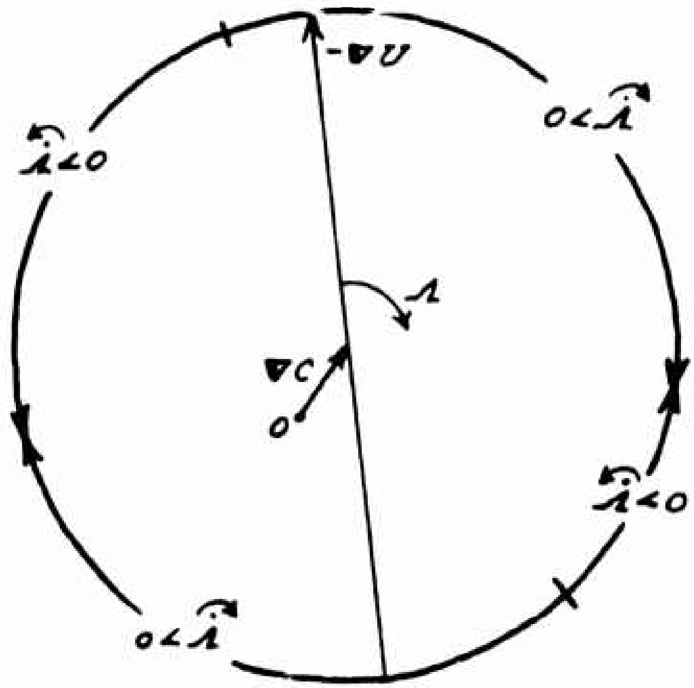


Figure 2. Stability of Ray Angles When The Gradient of Local Flow Speed Exceeds the Gradient of Local Speed of Sound.

We now write Equations (3) and (7) in difference form

$$\Delta x = \left[U - C\cos\Lambda\right]\Delta g \tag{8-a}$$

$$\Delta y = \left[C\sin\Lambda\right]\Delta g \tag{8-b}$$

and

$$\Delta\Lambda = -\left[\sin\Lambda\left(\frac{\partial C}{\partial x} - \cos\Lambda\frac{\partial U}{\partial x}\right) + \cos\Lambda\left(\frac{\partial C}{\partial y} - \cos\Lambda\frac{\partial U}{\partial y}\right)\right]\Delta g \tag{8-c}$$

where Δg represents an increment in disturbance travel time g, defined previously. To determine $\varphi(x, y, 0, x_o, y_o, 0)$ it is necessary to know a steady state distribution of $C(x, y)$, $U(x, y)$, and their derivatives at any point in the flow field over the wing and in the surrounding flow field in the plane of the wing. A means for establishing these is given in Section 5. Assume they are known. Then the procedure used is as follows:

1. Select any source point, on or off the wing, (x_o, y_o).

2. Select a series of initial ray angles, Λ_i, $i = 1, 2,$ ----.

3. Select an initial increment in disturbance travel time, Δg_o.

4. For each of the ray angles store $x^{(i)}$, $y^{(i)}$, $\sin \Lambda^{(i)}$, $\cos \Lambda^{(i)}$, and $\Delta g^{(i)}$, $i = 1, 2,$ ----.

   a. At $x^{(i)}$, $y^{(i)}$ compute and store $x^{(i)} = x^{(i)} + \Delta x^{(i)}/2$ and $y^{(i)} + y^{(i)} + \Delta y^{(i)}/2$, holding Λ constant.

   b. Iterate on $x_2{}^{(i)} = x^{(i)} + \Delta x^{(i)}/2$, $y_2{}^{(i)} = y^{(i)} + \Delta y^{(i)}/2$, and $\Delta\Lambda(x_2{}^{(i)}, y_2{}^{(i)})$ until they converge or exceed ten trials. In the latter case replace $\Delta g^{(i)}$ by $\Delta g^{(i)}/2$ and repeat the iteration. If they converge in three trials or less, replace $\Delta g^{(i)}$ by $2\Delta g^{(i)}$.

   c. Replace $x^{(i)}$ by $x_2{}^{(i)}$, $y^{(i)}$ by $y_2{}^{(i)}$, and return to a.

The solutions presented above are believed to be good approximations to the exact solutions for the following reasons:

1. For the case of a uniform flow they reduce to the proper linearized expressions.

2. The phase of the disturbance will be exact, although the amplitude may be slightly in error.

3. In an inner region in the immediate neighborhood of the source location (x_o, y_o, z_o) they approach the correct solution.

4. For a one-dimensional mean flow with M_ℓ approaching unity they reduce to Landahl's earlier solution (Reference 2).

5. In the limit of steady flow ($\omega = 0$), the solutions give results equivalent to the local linearization method of Spreiter and Alksne (Reference 4). This has been demonstrated by Rubbert (Reference 5).

6. Inasmuch as the proposed approximation only affects the receding part of the solution, the proper limiting solution for high frequencies (Reference 1), should always be obtained since then receding-wave effects are largely cancelled out due to the rapid phase variations.

This method gives reasonable results, i.e., reasonable based on a comparison with results obtained from the differential équation method. However, the ray paths did not conclusively show the existence of the focal point that the second method revealed.

Non-Linear Differential Equation Method

From Equations (3) we may write the slope of the ray path

$$\frac{dx}{dy} = \frac{M - \cos\Lambda}{\sin\Lambda} \tag{9}$$

and solving this equation for cos Λ, we get

$$\cos\Lambda = \frac{M \pm r\sqrt{r^2+1-M^2}}{1+r^2} \tag{10}$$

where $r = \frac{dx}{dy}$

The transmission time from source to receiving point is given by

$$\tau = \int \frac{ds}{V} \tag{11}$$

where the integration is taken along the path and

$$ds = \sqrt{1 + r^2}\quad dy \tag{12}$$

The velocity along the path is obtained from the vector sum of the two velocity components

$$V = C\sqrt{M^2 + 1 - 2M\cos\Lambda} \tag{13}$$

Substituting equations (12), (13), and (10) into equation (11) we have:

$$\tau = \int \frac{\sqrt{1+r^2}\,dy}{C\sqrt{M^2+1-2M\left[\frac{M \pm r\sqrt{r^2+1-M^2}}{1+r^2}\right]}}$$

which reduces to

$$\tau = \int \frac{(1+r^2)\,dy}{C\sqrt{M^2r^2 \mp 2Mr\sqrt{r^2+1-M^2} + r^2+1-M^2}} \tag{14}$$

The radicand in the denominator is a perfect square. Thus,

$$\tau = \int \frac{(1+r^2)\,dy}{C\left[Mr \mp \sqrt{r^2+1-M^2}\right]}$$

which reduces to

$$\tau = \int \frac{Mr \pm \sqrt{r^2+1-M^2}}{C\,(M^2-1)}\,dy \tag{15}$$

At this point we relate the local acoustic velocity, $C = C(x, y)$, to the local Mach number by imposing the condition of conservation of energy. For non-viscous flow, the total temperature is conserved. It is easily verified, that under this condition

$$\frac{c^2}{c_\infty^2} = \frac{5+M^2}{5+M_\infty^2} \tag{16}$$

where $\gamma = 1.4$, for a diatomic gas, has been used. Substituting Equation (16) into Equation (15), we get

$$g = \frac{1}{c_\infty\sqrt{5+M_\infty^2}} \int \frac{\sqrt{5+M^2}\left[Mr \pm \sqrt{r^2+1-M^2}\right]}{(M^2-1)}\, dy \tag{17}$$

where the upper sign applies to receding waves and the lower sign to advancing waves. Equation (17) contains all the elements for the solution. However, the integrand is a function of x, y, and dx/dy. This equation may be written in symbolic form

$$g = \int_{y_0}^{y_1} F\left(x, y, \frac{dx}{dy}\right) dy$$

which suggests the use of Euler's equation to find the minimum time g, for the disturbance to travel to a field point (x_1, y_1)

$$\frac{d}{dy}\frac{\partial F}{\partial r} - \frac{\partial F}{\partial x} = 0 \tag{18}$$

In order to simplify the notation, we set

$$F = \frac{\delta\,(Mr \pm \beta)}{M^2-1}$$

where $\quad \delta = \delta(x, y) = \sqrt{5 + M^2}$

$\qquad\quad \beta = \beta(x, y, r) = \sqrt{r^2 + 1 - M^2}$

and r has been previously defined. We will need

$$\frac{\partial F}{\partial x} = \frac{\delta}{M^2-1}\left[rM_x \mp \frac{MM_x}{\beta}\right] + \frac{Mr \pm \beta}{(M^2-1)^2}\left[(M^2-1)\frac{MM_x}{\delta} - 2\delta MM_x\right]$$

$$\frac{d}{dy}\left(\frac{\partial F}{\partial r}\right) = \frac{\delta}{M^2-1}\left[\frac{dM}{dy} \pm \frac{\beta\frac{dr}{dy} - r\frac{d\beta}{dy}}{\beta^2}\right]$$

$$+\left(M \pm \frac{r}{\beta}\right)\left[\frac{(M^2-1)\frac{d\delta}{dy} - 2\delta M\frac{dM}{dy}}{(M^2-1)^2}\right]$$

Then, making use of the relationships

$$\frac{dM}{dy} = rM_x + M_y$$

$$\frac{d\beta}{dy} = \frac{1}{\beta}\left[r\frac{dr}{dy} - rMM_x - MM_y\right]$$

$$\frac{d\delta}{dy} = \frac{1}{\delta}\left[rMM_x + MM_y\right],$$

solving for dr/dy, and combining terms, we get

$$\frac{dr}{dy} = \frac{1}{\delta^2(M^2-1)}\left\{\left[\frac{-M(M^2+1)r}{M^2-1} \mp \frac{\beta(7M^2+5)}{M^2-1}\right]r^2 + \left[2M(M^2+8)r \pm \beta(7M^2+5)\right]\right\}M_y + \left\{\frac{M}{\delta^2}(r^2+6)\right\}M_x \tag{19}$$

Equation (19) is a second order, second degree differential equation of the form

$$\frac{d^2x}{dy^2} = f\left(x, y, \frac{dy}{dx}\right)$$

It is second degree because β represents a radical. However, it can be solved numerically by any of the standard repetitive processes. We employed a fourth order Runge-Kutta procedure.

There are certain difficulties that arise in the numerical evaluation of Equation (19). These are first listed and interpreted and then equations used to surmount them are presented.

(1) Along some ray paths dx/dy becomes infinite even when the Mach number is not equal to one.

(2) Equation (19) is singular at Mach number = 1.0.

(3) In the supersonic region, signals sometimes become trapped on the local Mach line. This happens when $\cos\Lambda = 1/M$. Signals tend to gravitate to this condition. Such trapped signals cannot then cross the sonic line. They approach the sonic line as a limit, and are cancelled out there.

To overcome the difficulty listed in Item (1), it is necessary to use x instead of y as the independent variable. This is done by applying the equation

$$\frac{d^2y}{dx^2} = \frac{-1}{\left(\frac{dx}{dy}\right)^3}\frac{d^2x}{dy^2} \tag{20}$$

It is convenient here to introduce some new notation. Re-write equation (19) in the form

$$x'' = \frac{1}{AB}\left\{-\frac{M}{B}(M^2+11)x'^3 + 2M(M^2+8)x' \mp (7M^2+5)\frac{R_1^3}{B}\right\}M_y + \frac{M}{A}\left\{x'^2+6\right\}M_x \tag{21}$$

where the new notation, together with some other notation which will be used later, is defined as follows

$$\begin{array}{lll} x' = \dfrac{dx}{dy} & A = M^2+5 & M_x = \dfrac{\partial M}{\partial x} \\ y' = \dfrac{dy}{dx} & B = M^2-1 & M_y = \dfrac{\partial M}{\partial y} \\ R_1 = \sqrt{x'^2-(M^2-1)} & \beta = \sqrt{B} & \\ R_2 = \sqrt{1-y'^2(M^2-1)} & E = c_\infty\sqrt{5+M_\infty^2} & \end{array} \tag{22}$$

Substituting Equation (20) into Equation (21), we get

$$y'' = \frac{1}{AB}\left\{\frac{M}{B}(M^2+11) - 2M(M^2+8)y'^2 \pm \frac{R_2^3}{B}(7M^2+5)\right\}M_y - \frac{My'}{A}(6y'^2+1)M_x \quad ; \quad 0 \leq y' \tag{23-a}$$

$$y'' = \frac{1}{AB}\left\{\frac{M}{B}(M^2+11) - 2M(M^2+8)y'^2 \mp \frac{R_2^3}{B}(7M^2+5)\right\}M_y - \frac{My'}{A}(6y'^2+1)M_x \quad ; \quad y' < 0 \tag{23-b}$$

The limiting form of Equation (20) at M = 1 is:

$$x''\Big|_{M=1.0} = \frac{1}{2A}\left\{2x'^3 + x' + \frac{9}{x'}\right\}M_y + \frac{1}{A}(x'^2+6)M_x$$

In the supersonic region, when the signal is trapped on the local Mach line, and

$$\cos\mu = \frac{1}{M}, \quad \sin\mu = \frac{\sqrt{1-M^2}}{M}, \quad \text{and} \quad |x'| = \beta$$

equation (20) reduces to

$$x'' = M\left(\frac{M_y}{x'} + M_x\right)$$

A complete set of equations, together with their areas of applicability, will now be outlined.

Complete Set of Equations where Y is the Independent Variable

$$x'' = \frac{1}{AB}\left\{\frac{-M}{B}(M^2+11)x'^3 + 2M(M^2+8)x' \mp (7M^2+5)\frac{R_1^3}{B}\right\}M_y + \frac{M}{A}(x'^2+6)M_x \tag{24}$$

$$\frac{dt}{dy} = \frac{1}{E}\,\frac{\sqrt{5+M^2}\,(Mx' \pm R_1)}{M^2-1} \tag{25}$$

$$x''\Big|_{M=1.0} = \frac{1}{2A}\left\{2x'^3 + x' + \frac{9}{x'}\right\}M_y + \frac{M}{A}(x'^2+6) \tag{26}$$

$$\frac{dt}{dy}\Big|_{M=1.0} = \frac{\sqrt{6}}{2E}\left(x' + \frac{1}{x'}\right) \tag{27}$$

$$x''\Big|_{|x'|=\beta} = M\left(\frac{M_y}{x} + M_x\right) \tag{28}$$

$$\frac{dt}{dy}\Big|_{|x'|=\beta} = \frac{M\sqrt{5+M^2}}{E\,x'} \tag{29}$$

A complete set of equations were also developed using x as the independent variable. However, for the sake of brevity, and since they are obtained by a simple change of variable, they will not be listed here. Equations (26) and (27) apply where an advancing ray path crosses the sonic line, and equations (28), (29) apply where a ray path, in the supersonic region, becomes trapped on the local Mach line. It remains to describe the regions of applicability of the upper and lower signs of equations (24) and (25). In what follows, "right branch" will be specified where $(0 < \lambda < \pi)$ and left branch will be specified if $(-\pi < \lambda < 0)$. Here λ is the local value along the ray path. The end points are not specified because for these points we use x as the independent variable.

The upper sign is used for

(1) Subsonic, left branch

(2) Supersonic, receding, right branch

(3) Supersonic, advancing, left branch

The lower sign is used for

(1) Subsonic, right branch

(2) Supersonic, receding, left branch

(3) Supersonic, advancing, right branch

THE NON-UNIFORM FLOW FIELD

In the application of each of the methods contained in this report, it is necessary to know certain of the properties of the transonic flow field on, and in the neighborhood of, the wing. Figures 3 and 4 show the distributions of local flow speeds and sonic speeds over a 65° delta wing model in a wind tunnel in which the Mach number was 1.04 (taken from Reference 6). Speeds were computed from steady state pressure data at 27 points on the wing. The figures are intended only to show the general characteristics of the flow, such as: (1) The local sonic line shifts aft with distance from the centerline but crosses the leading edge inboard of the tip, (2) Mach number variations in both the streamwise and spanwise directions must be considered and cannot be considered to be linear, and (3) Separated flow is indicated over the aft and inboard portion of the wing. To consider the last of these characteristics is beyond the scope of this study. However, the first two are amenable to analysis using available theories and techniques.

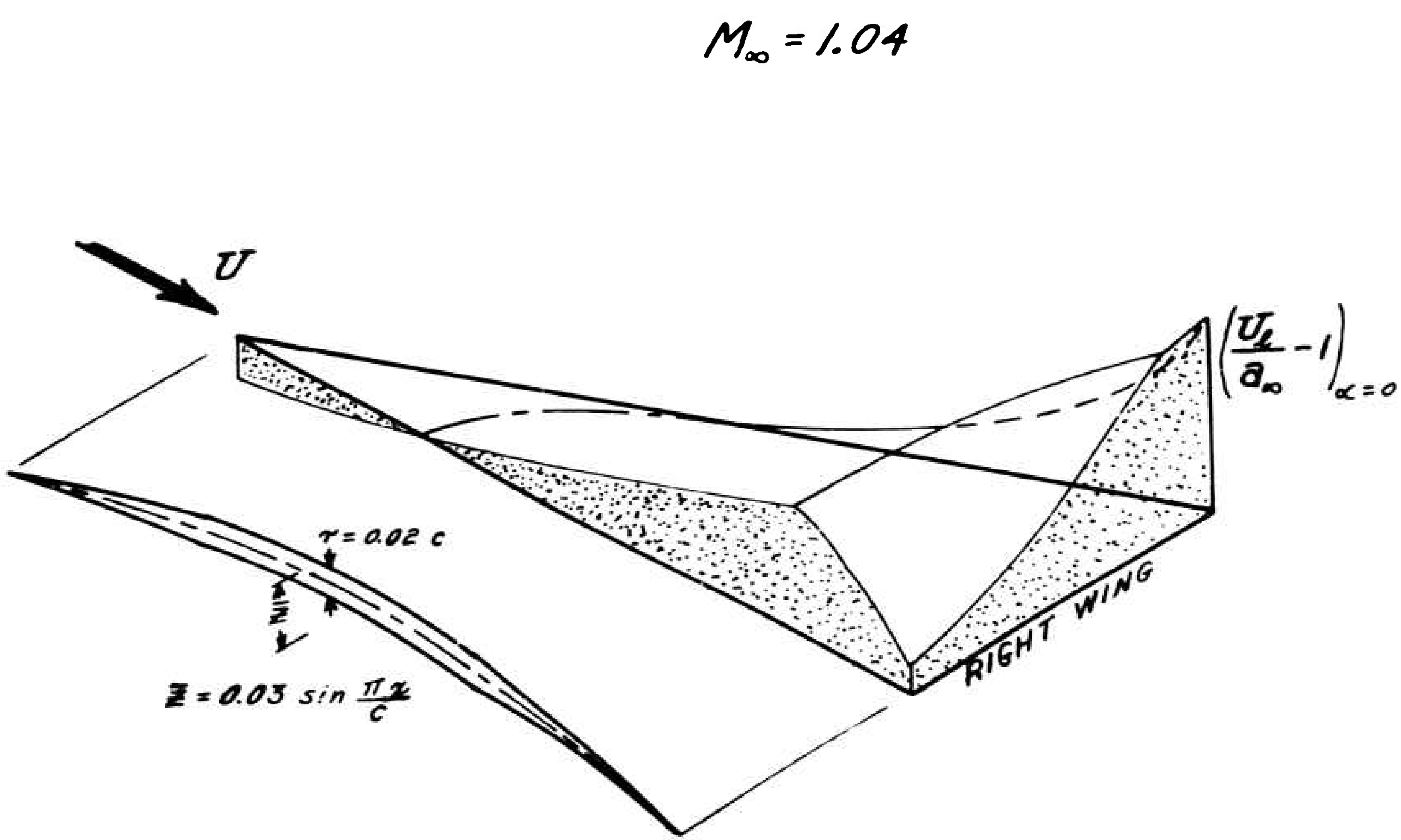


Figure 3. Local Flow Distribution on a 65° Δ at a Transonic Speed

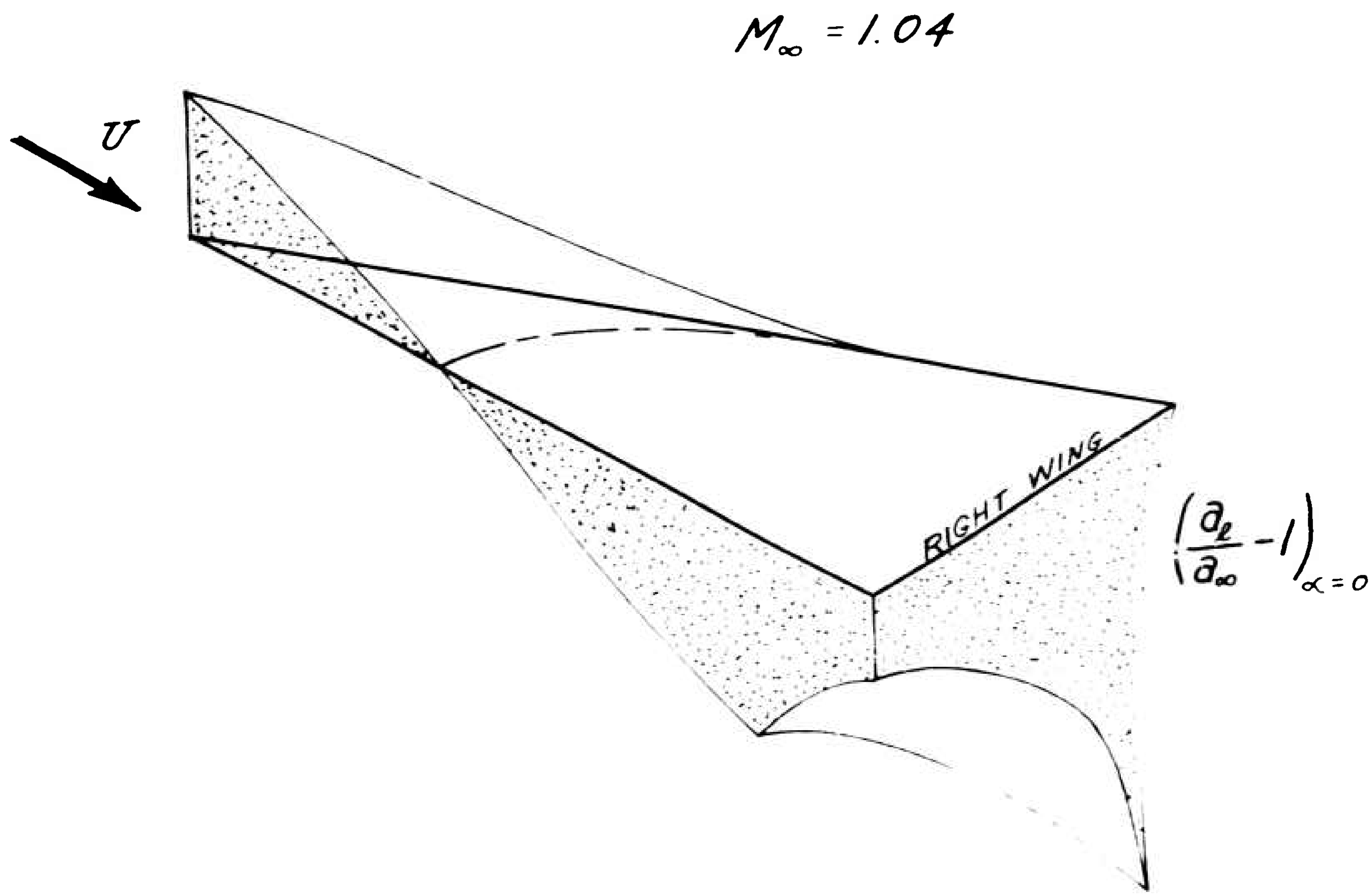


Figure 4. Sonic Speed Distribution on a 65° Δ at a Transonic Speed

Mach number distributions over areas off the wing were computed from an approximate theoretical solution of the flow field that matched pressure distributions on the wing. In order to avoid a discontinuity at the juncture of the two regions, a small transition region was defined over which the two functions were joined by a numerical smoothing technique.

Let:

$M_L = M_L(x,y) =$ Mach number

$\varphi = \varphi(x,y) =$ Perturbation potential

$\tau = T/C(x,y) =$ Thickness ratio

For a steady-state, non-lifting flow

$$(1-M_L^2)\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \tag{30}$$

and

$$\varphi_z(x,y,0^+) = \pm\tau f_x(x,y) \tag{31}$$

Where $f(x,y)$ is a function describing the variation of the surface from the mean.

Using parametric differentiation with respect to τ, (Reference 5),

$$g = g(x,y) = \frac{\partial\varphi}{\partial\tau}$$

Equation (30) becomes:

$$\frac{\partial}{\partial x}\left\{\left[1-M_L^2\right]g_x\right\} + g_{yy} + g_{zz} = 0 \tag{32}$$

$$g_z(x,y,0^+) = \pm f_x(x,y)$$

After having obtained the solution of equation (32), the local Mach number distribution is obtained by relating local Mach number to the coefficient of pressure, (C_p). Starting with the following basic relations:

Let $\quad u = \dfrac{V_L - V_\infty}{V_\infty}$

then $\quad u = \dfrac{1}{V_\infty}\dfrac{\partial\varphi}{\partial x} = -C_p/2 \qquad (33)$

$$a^2 + \frac{1}{2}(\gamma-1)q^2 = \text{constant} \tag{34}$$

where $\quad q = V_\infty$ at infinity

$q = V_\infty(1+u)$ elsewhere

a = speed of sound

We have: $$a_\infty^2 + \tfrac{1}{2}(\gamma-1)v_\infty^2 = a_L^2 + \tfrac{1}{2}(\gamma-1)v_\infty^2(1+u)^2$$
$$v_\infty^2(1+u)^2 \cong v_\infty^2(1+2u)$$
$$a_L^2 \cong a_\infty^2 - (\gamma-1)v_\infty^2 u$$
using equation (33)
$$a_L^2 \cong a_\infty^2\left[1+\tfrac{1}{2}(\gamma-1)M_\infty^2 C_p\right]$$

The coefficient of pressure, C_p is of order (.1), and M is O(1.). Therefore, to sufficient accuracy,
$$a_L \cong a_\infty\left[1+\tfrac{1}{4}(\gamma-1)M_\infty^2 C_p\right]$$
$$V_L = V_\infty(1+u) = v_\infty\left(1-\tfrac{1}{2}C_p\right)$$
and from these relations:
$$M_L \cong \frac{M_\infty\left(1-\frac{1}{2}C_p\right)}{1+\frac{1}{4}(\gamma-1)M_\infty^2 C_p}$$
Noting again the order of M_∞ and C_p, to sufficient accuracy,
$$M_L \cong M_\infty\left(1-\tfrac{1}{2}C_p\right)\left[1-\tfrac{1}{4}(\gamma-1)M_\infty^2 C_p\right]$$
or
$$M_L \cong M_\infty\left[1-\frac{\gamma+1}{4}C_p\right] \qquad (35)$$

Equation (35) is the expression that was used to relate local Mach number to C_p on regions off the wing.

A solution of equation (32), using the results of equation (35) was worked out for a special configuration. The special wing configuration is depicted in figure (5).

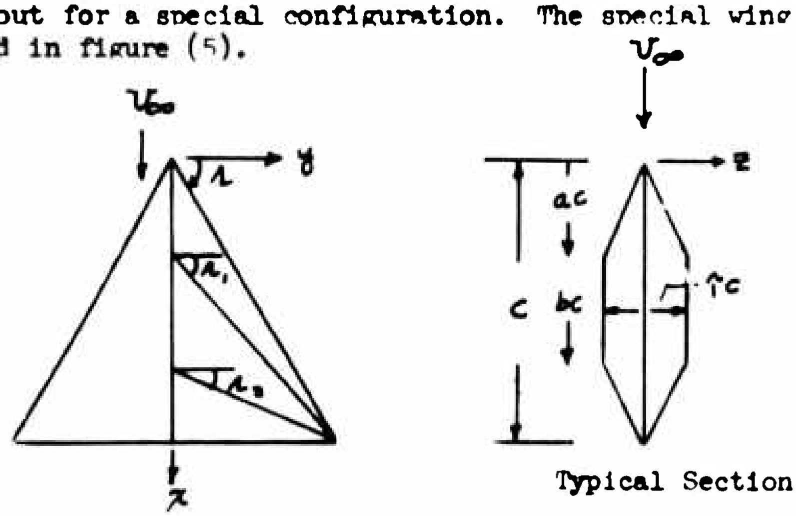


Fig. 5. A Thin Wing In Rectilinear Flight

The solution is:
$$\begin{aligned} C_p(x,y)-C_p(x,0) = &-2f\left[|y-s|^{\varepsilon}+|y+s|^{\varepsilon}-2|s|^{\varepsilon}\right] \\ &-2f_1\left[|y-s_1|^{-\varepsilon_1}+|y+s_1|^{-\varepsilon_1}-2|s_1|^{-\varepsilon_1}\right]H(x-a) \\ &-2f_2\left[|y-s_2|^{-\varepsilon_2}+|y+s_2|^{-\varepsilon_2}-2|s_2|^{-\varepsilon_2}\right]H(x-b) \end{aligned} \qquad (36)$$

where $H(x)$ is a step function.

$$\frac{\partial \Delta C_p}{\partial x} = -\frac{2f\varepsilon}{\tan\Lambda}\left[-|y-s|^{\varepsilon-1} + |y+s|^{\varepsilon-1} - 2|s|^{\varepsilon-1}\right]$$
$$+\frac{2f_1\varepsilon_1}{\tan\Lambda_1}\left[-|y-s_1|^{-\varepsilon_1-1} + |y+s_1|^{-\varepsilon_1-1} - 2|s_1|^{-\varepsilon_1-1}\right]H(x-a)$$
$$+\frac{2f_2\varepsilon_2}{\tan\Lambda_2}\left[-|y-s_2|^{-\varepsilon_2-1} + |y+s_2|^{-\varepsilon_2-1} - 2|s_2|^{-\varepsilon_2-1}\right]H(x-b) \qquad (37)$$

$$\frac{\partial \Delta C_p}{\partial y} = -2f\varepsilon\left[|y-s|^{\varepsilon-1} + |y+s|^{\varepsilon-1}\right]$$
$$+2f_1\varepsilon_1\left[|y-s_1|^{-\varepsilon_1-1} + |y+s_1|^{-\varepsilon_1-1}\right]H(x-a)$$
$$+2f_2\varepsilon_2\left[|y-s_2|^{-\varepsilon_2-1} + |y+s_2|^{-\varepsilon_2-1}\right]H(x-b) \qquad (38)$$

WHERE:

$$f = \frac{\cos^2\Lambda}{\sin\Lambda}, \quad \varepsilon = \frac{\tau}{2\pi a\cos\Lambda}, \quad s = x/\tan\Lambda$$
$$f_1 = \frac{\cos^2\Lambda_1}{\sin\Lambda_1}, \quad \varepsilon_1 = \frac{\tau}{2\pi a\cos\Lambda_1}, \quad s_1 = (x-a)/(1-a)\tan\Lambda_1,$$
$$f_2 = \frac{\cos^2\Lambda_2}{\sin\Lambda_2}, \quad \varepsilon_2 = \frac{\tau}{2\pi a\cos\Lambda_2}, \quad s_2 = (x-b)/(1-b)\tan\Lambda_2$$
$$\tan\Lambda_1 = (1-a)\tan\Lambda, \quad \tan\Lambda_2 = (1-b)\tan\Lambda$$

After determining a distribution of C_p and its derivatives from equations (36), (37), and (38), the Mach number distribution, with its derivatives, is computed from equation (35).

DESCRIPTION OF THE COMPUTER PROGRAM

The equations for the ray paths are solved in the following manner: Let the independent variable be y and

$$V_1 = \frac{dx}{dy}$$
$$V_2 = x$$
$$V_3 = t$$

Then

$$\frac{dV_1}{dy} = f_1(V_1, V_2, y)$$
$$\frac{dV_2}{dy} = V_1$$
$$\frac{dV_3}{dy} = f_3(V_1, V_2, y)$$

These three simultaneous differential equations are solved in a step-by-step manner by use of a standard "SHARE" subroutine which is based on the Runge Kutta method. When dx/d_y becomes greater than one, a variable change takes place in the program, and x becomes the independent variable.

A signal (in the supersonic region) is considered "trapped" on the local Mach line when

$$\left| x'^2 - (M^2 - 1) \right| \leq E1$$

When, for this trapped signal, $(M-1) < E2$, the integration stops and a new ray line is started. This logical flow is shown in the chart on page 21.

The values of λ_0 used in the program are determined by the parameter (NLA). If (NLA) is an odd integer, it will be rounded down in the program to an even integer. Values of λ_0 vary from zero to π and from zero to $-\pi$ in an arithmetic progression.

Computation of a ray path (other than for a "trapped signal") ceases under the following conditions:

$$x \leq 0$$
$$1 \leq x$$
$$|YMAX| \leq |y|$$
$$NMAX \leq NCNT$$

where NCNT is the number of points on the ray path already computed. This logical flow is shown in the chart on page 22.

Subroutine DERIV computes the appropriate derivatives.

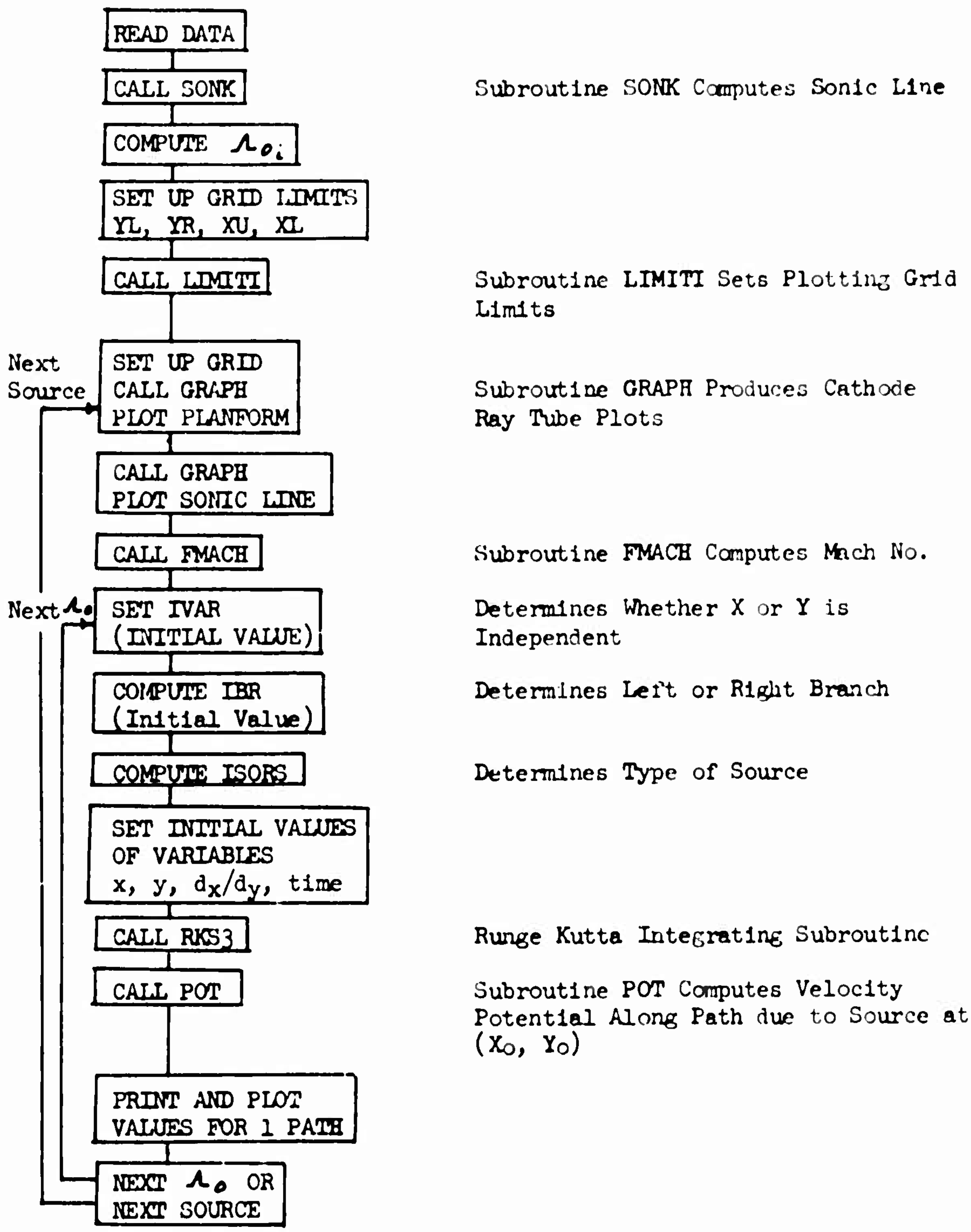
Subroutine CNTRL accomplishes variable changes, stores local values in appropriate locations for later printing, and performs exit tests.

Subroutine FMACH computes the local Mach number and the partial derivatives of the Mach number.

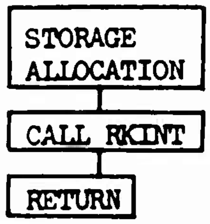
Subroutine SONK computes coordinates on the planform where $M = 1$.

Sample data sheets with numbers which have been used in a computer run are in Appendix II. The output sheets are included. The output format is self-explanatory, with the exceptions of certain test words that are printed out at the beginning of the plots for each ray-path. Definitions for these words can be found in the comment statements at the beginning of the listing in Appendix I. The values listed for these test words apply to the last point plotted for the ray-path.

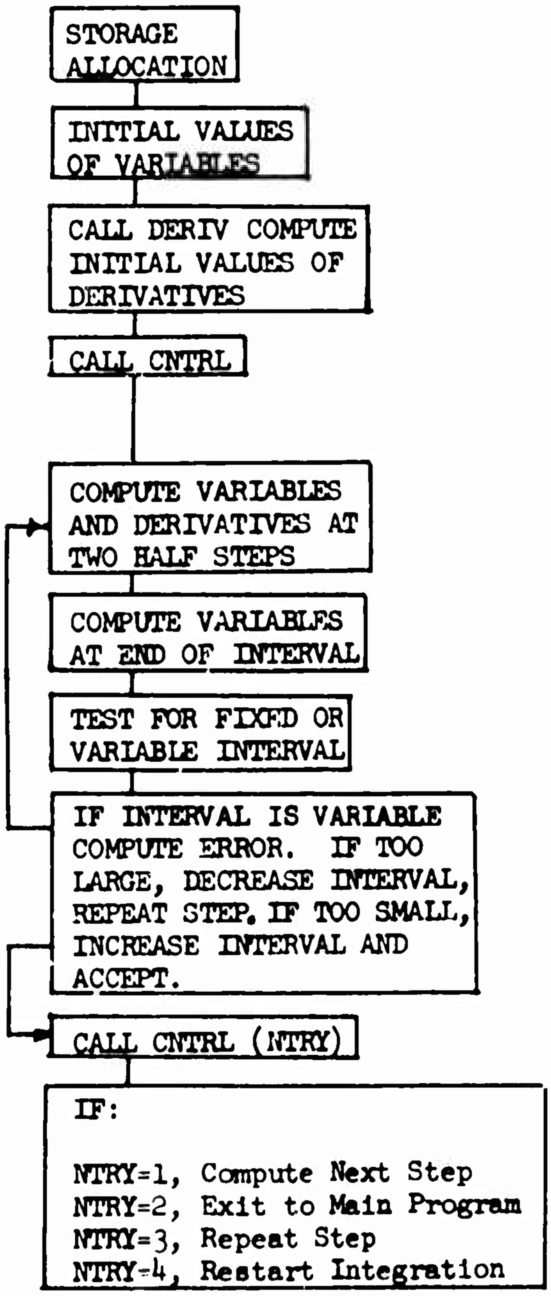
MAIN PROGRAM



SUBROUTINE RKS3



SUBROUTINE RKINT



Subroutine DERIV Computes Derivatives

Subroutine CNTRL Executes Variable Changes, Stores Current Values, Executes Exit Tests

This Loop Calls DERIV 8 Times

NTRY is Re-set in CNTRL

DISCUSSION OF RESULTS

This report contains two methods for calculating the velocity potential along sonic ray lines emanating from any point in a non-uniform flow field, i.e., one that varies from locally subsonic to supersonic speeds. Both methods apply to pulses emitted by sources or doublets. It has been demonstrated that both methods yield nearly identical ray paths and times of transmission. Those presented were obtained using the second method.

Figures 6 through 13 show ray paths of acoustic signals emanating from various points in a non-uniform transonic flow field. The reader may want to try his hand at tracing one of the ray paths in a region of interest such as near a leading edge. If so, it should be helpful to recall the discussion starting with Equation (7), through the difference equations of the path, Equation (8), and to the end of that section. An analysis of the differential equation of the path, Equation (24) should also be helpful. These show, for instance, that where the Mach number is constant the curvature of the ray path is zero; for a given Mach number and slope of ray path the curvature is proportional to rate of change of Mach number along the path. Figures 6, 7, 9, and 10 conclusively show that when the variation in Mach number is parabolic in the chordwise and spanwise directions focal pointsexist, both in subsonic and supersonic portions of the flow. None of the present theories accounts for the corresponding multiple crossings of the acoustic wave front. Figures 9 and 12 show acoustic signals traveling from regions of supersonic flow to regions of subsonic flow. This can occur, of course, only when the sonic line is swept downstream. Figures 9 and 12 also show rays that have been trapped on the Mach wave, travel outward to the sonic line where the spanwise slope of the ray path becomes zero, and are cancelled there. A study of the ray paths that cross the leading edge shows that in practical applications it is correct to assume they do not return.

These results permit the formulation of a numerical procedure. A box method is outlined in Appendix III. It establishes velocity potentials at all box centers on an aerodynamic surface and the corresponding generalized forces.

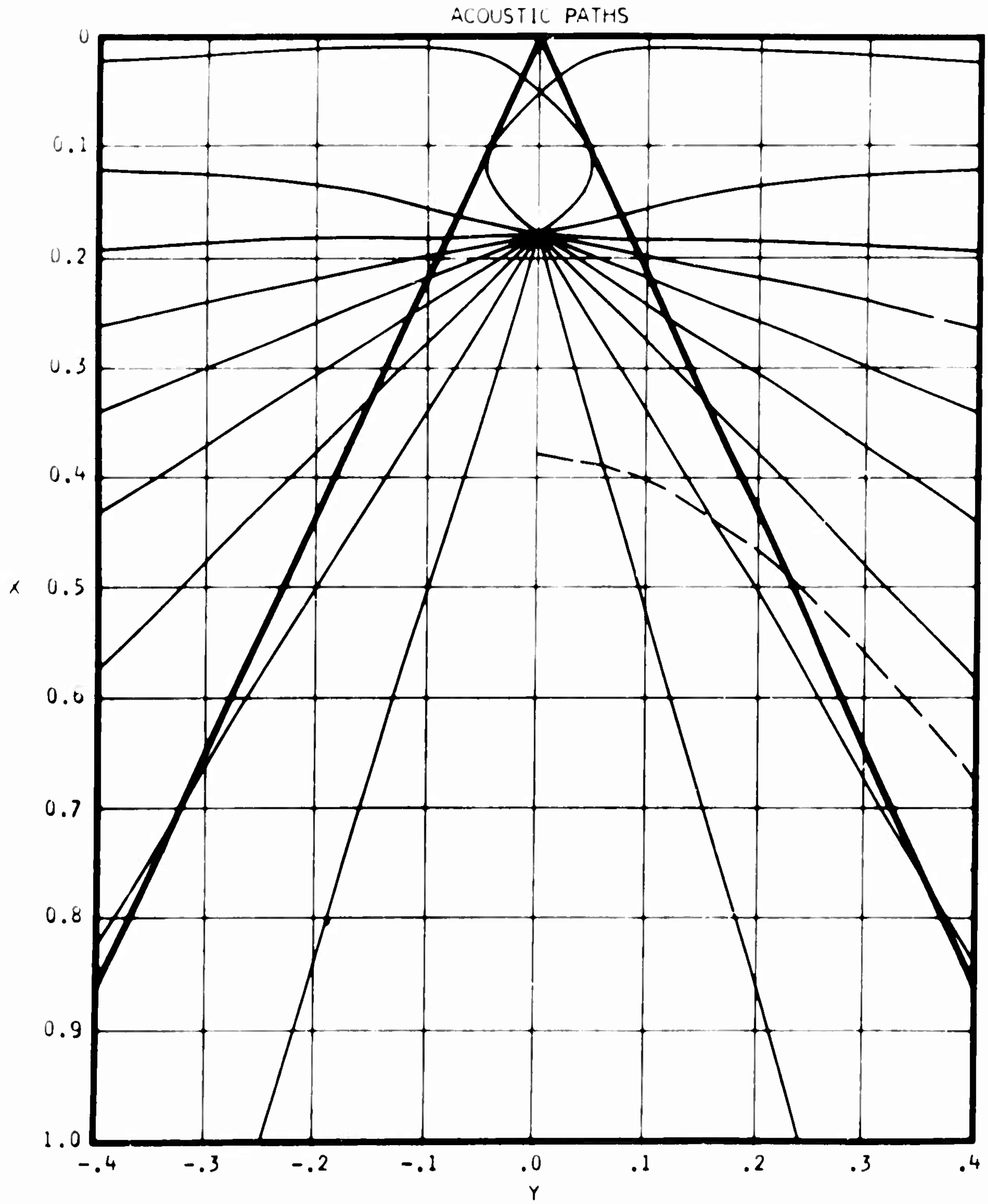


Figure 6. Ray Paths for a Source or Doublet at (0.18c, 0.0)

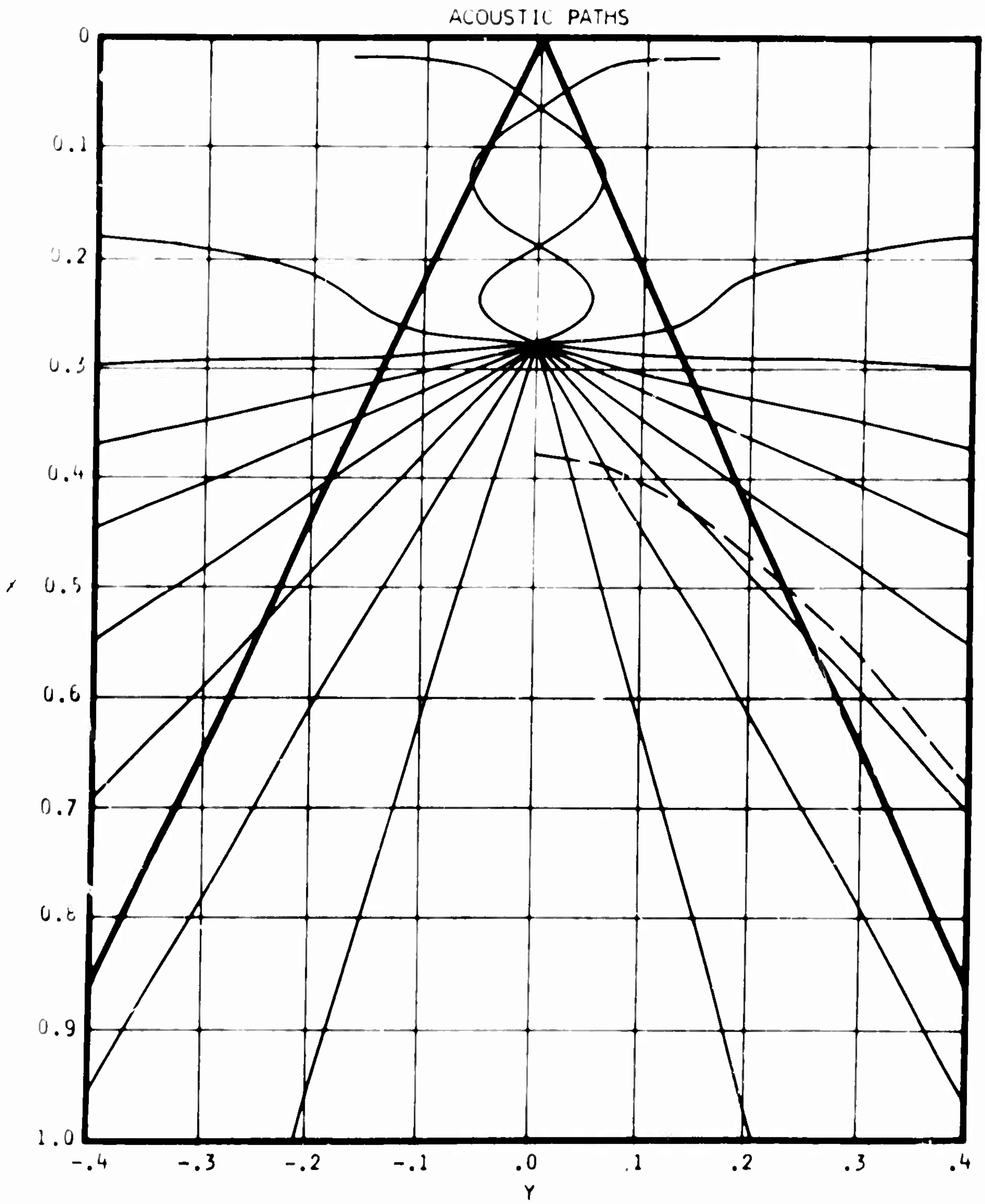


Figure 7. Ray Paths for a Source or Doublet at (0.28c, 0.0)

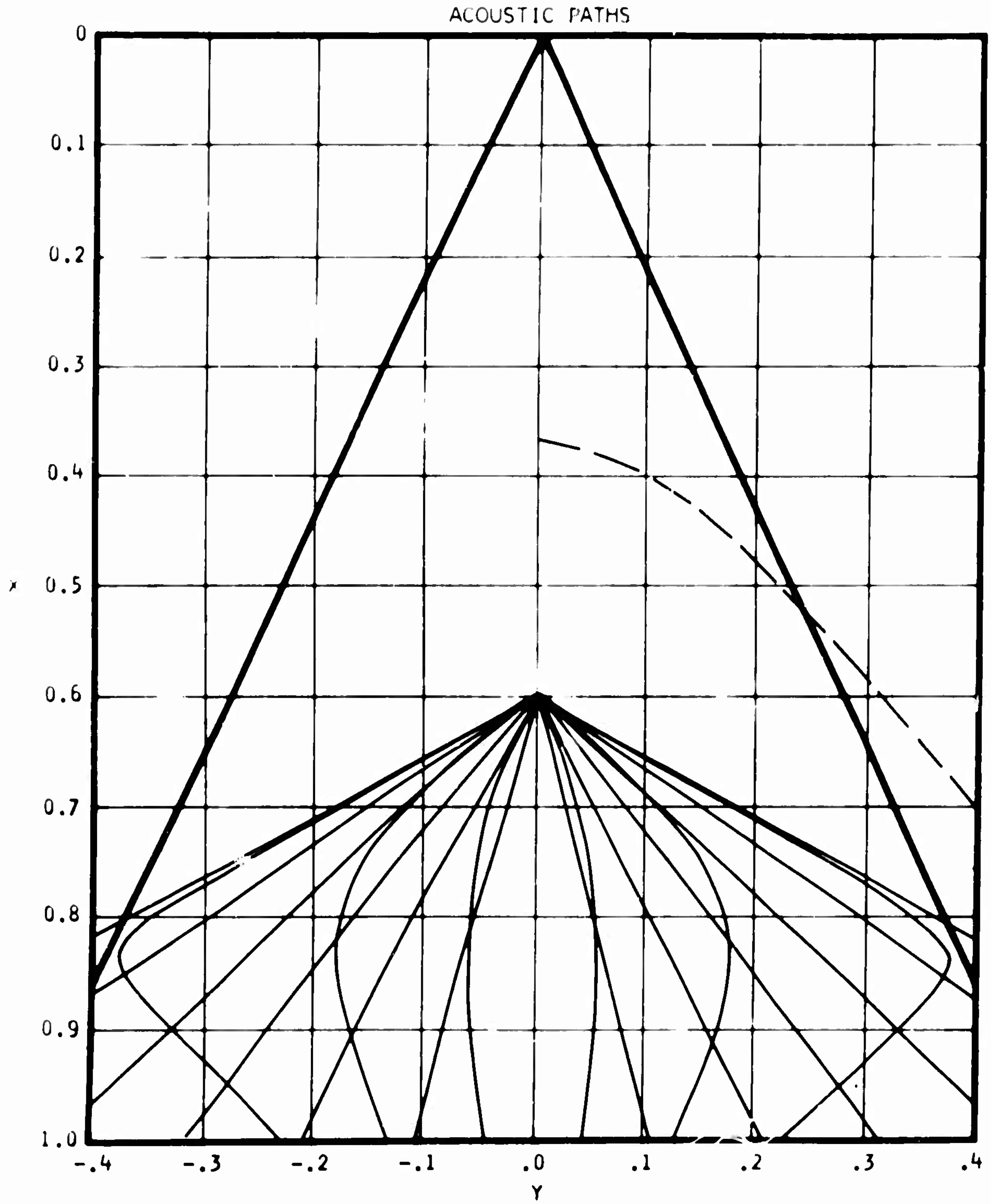


Figure 6. Ray Paths for a Source or Doublet at (0.6c, 0)

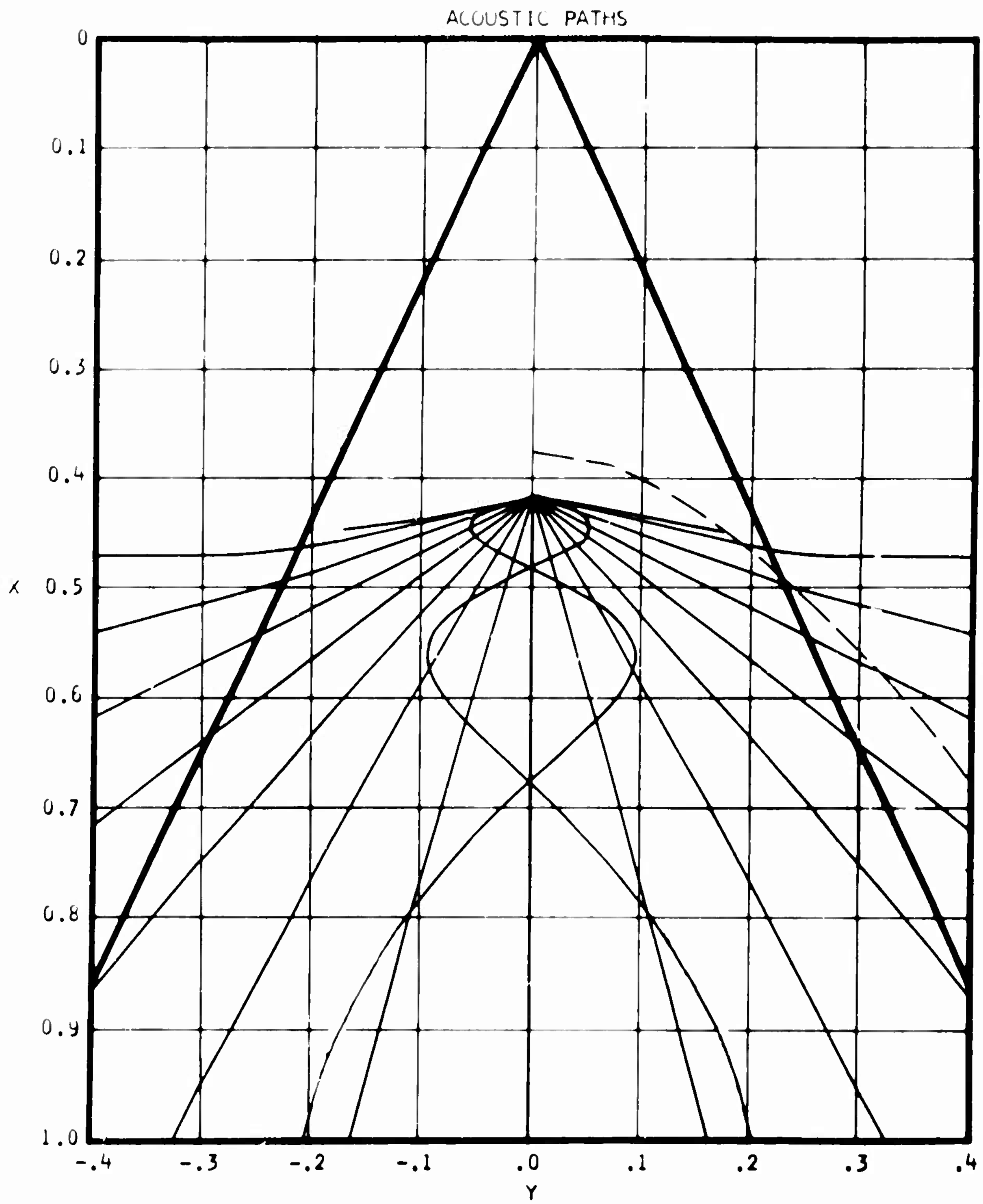


Figure 9. Ray Paths for a Source or Doublet at (0.42c, 0.0)

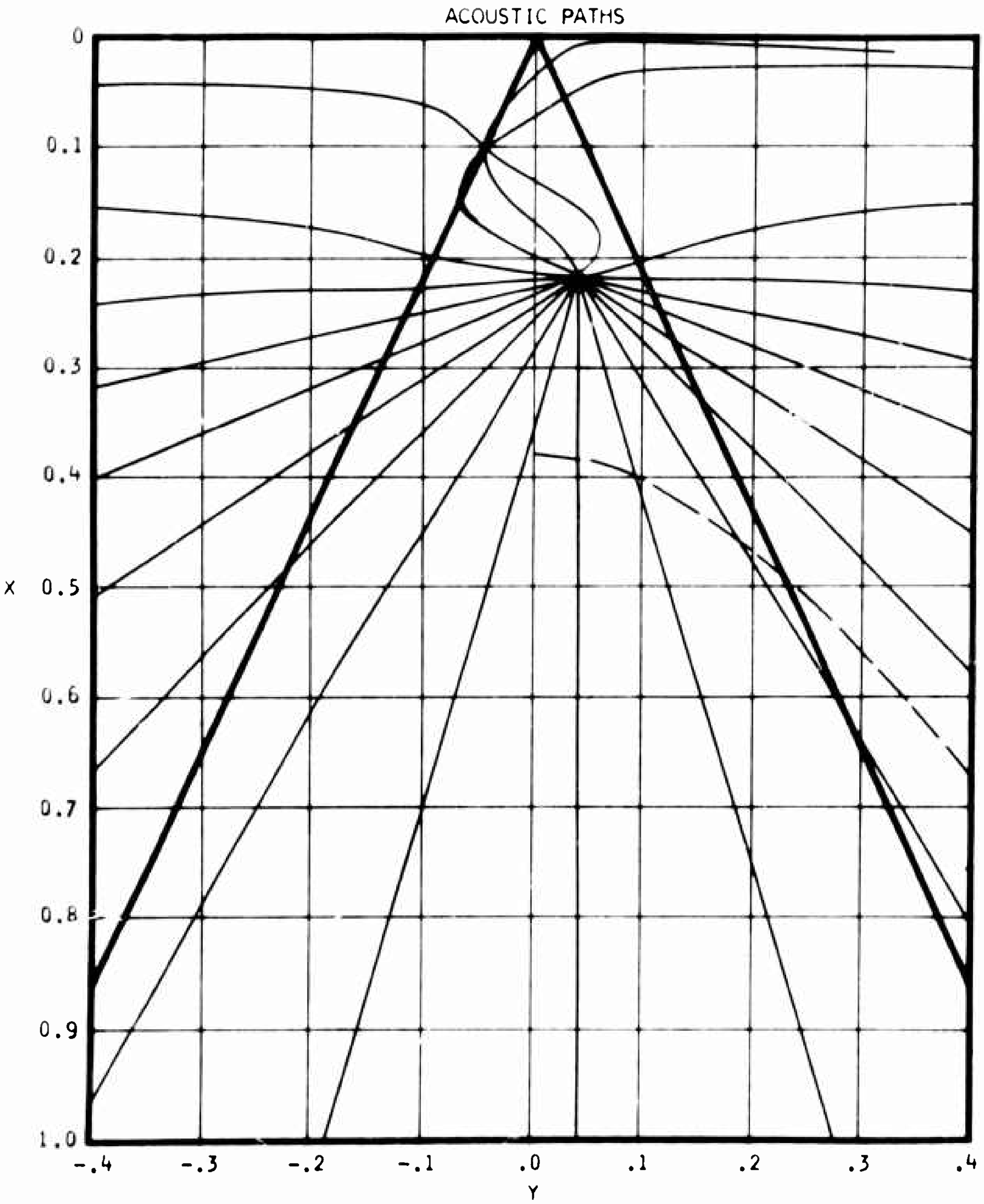


Figure 10. Ray Paths for a Source or Doublet at (0.22c, 0.04c)

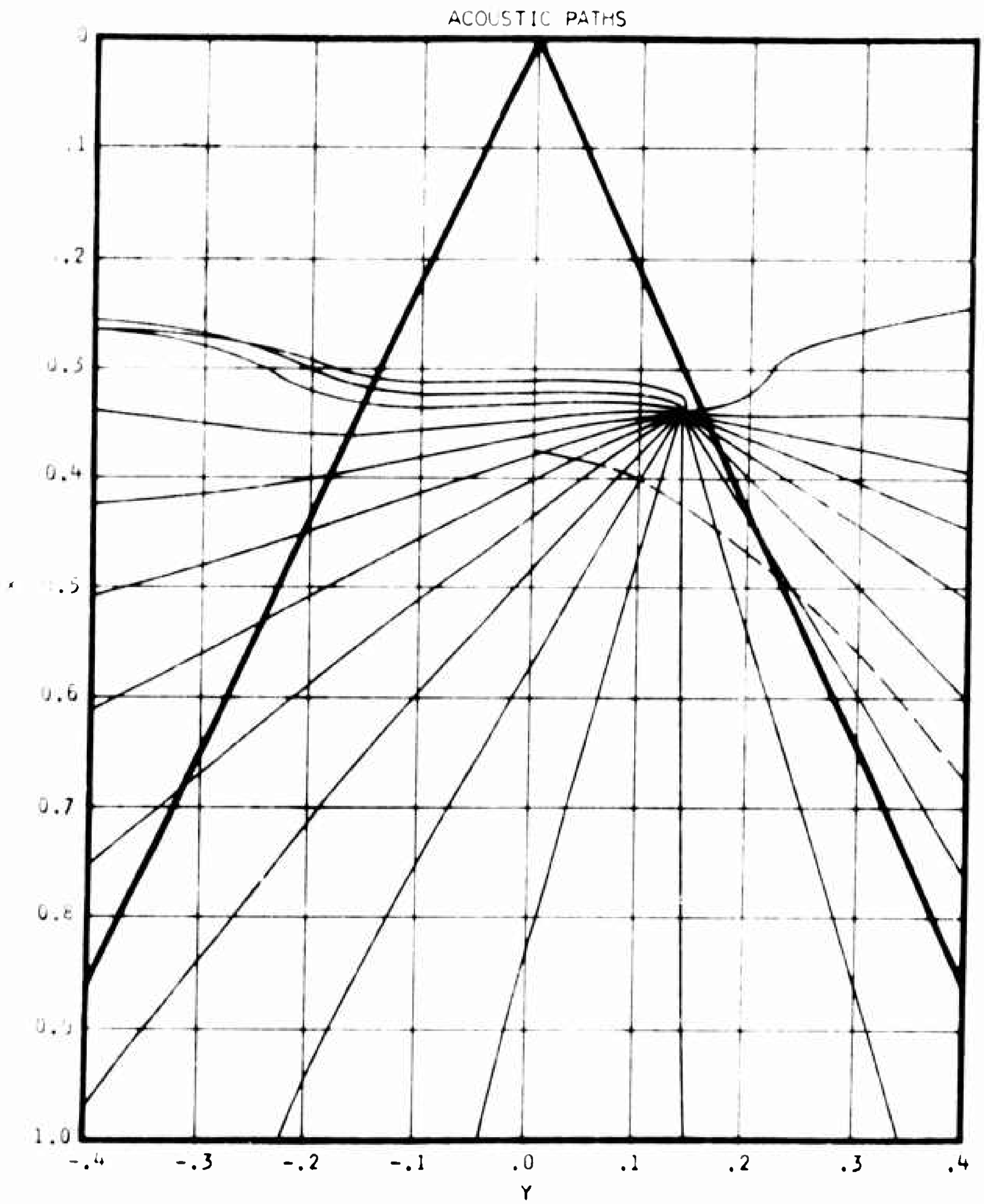


Figure 11. **Ray Paths for a Source or Doublet at** (0.34c, 0.14c)

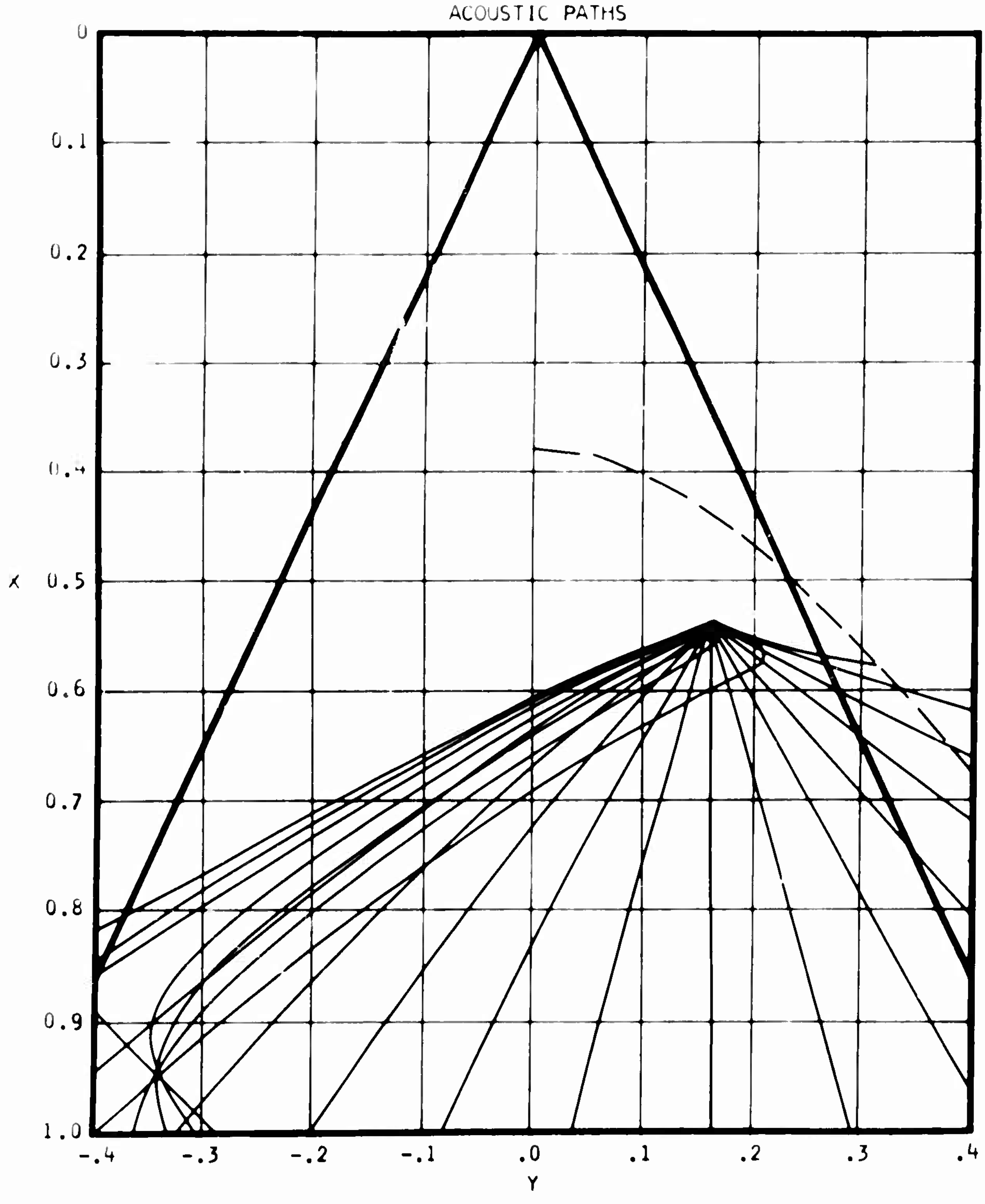


Figure 12. Ray Paths for a Source or Doublet at (0.54c, 0.16c)

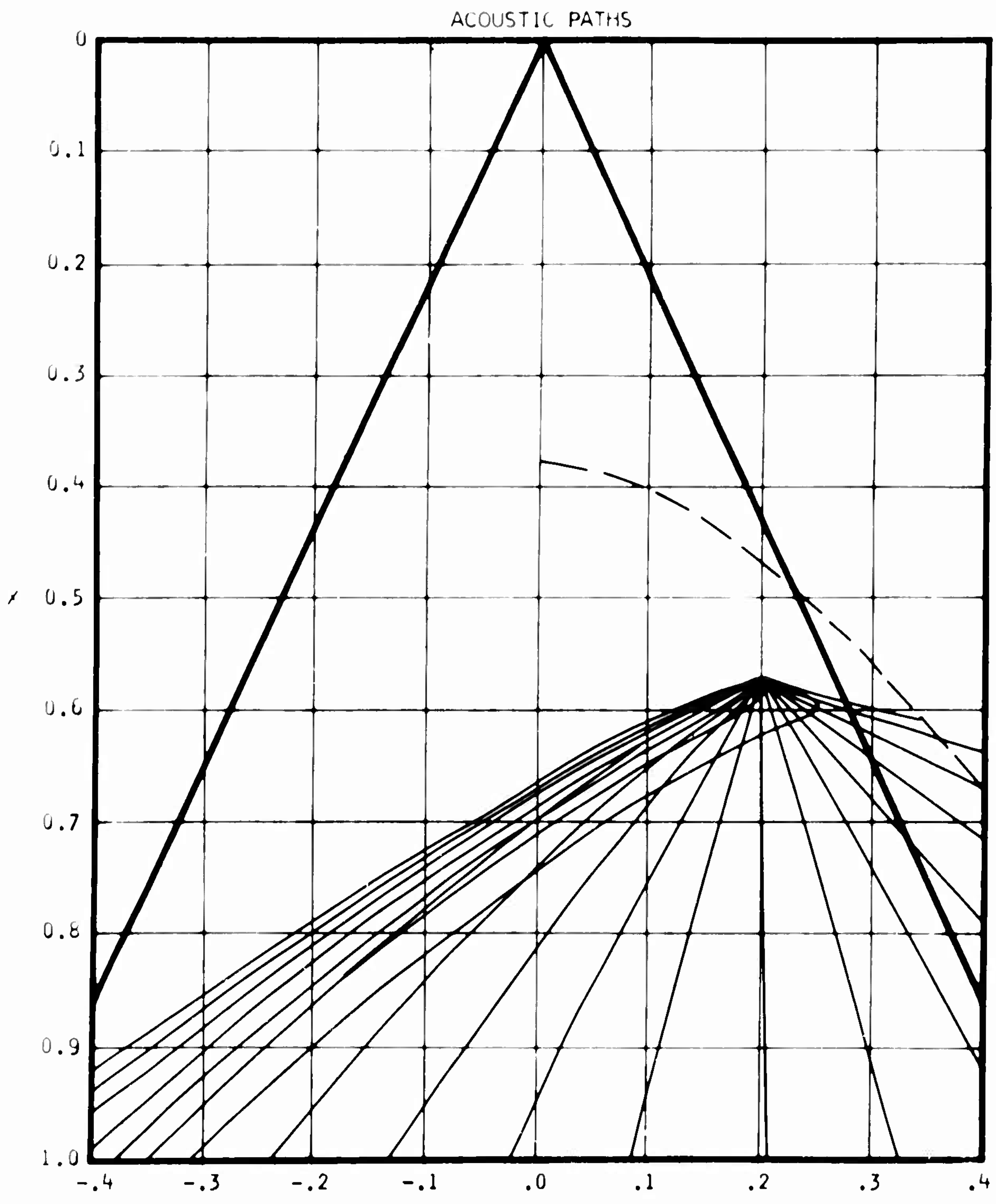


Figure 13. Ray Paths for a Source or Doublet at (0.57c, 0.20c)

CONCLUSIONS AND RECOMMENDATIONS

Two methods have been outlined in detail, and one of them has been completely mechanized for calculating the velocity potentials along acoustic ray paths emanating from any point in a non-uniform transonic flow field over a lifting surface. The one mechanized gives the ray path and velocity potential for the minimum time of travel from the source point to the field point.

To calculate pressures over the planform and generalized forces, it will be necessary to develop a procedure for calculating the velocity potential at an arbitrary point due to a sheet of sources, covering the wing surface, and the flow field in the plane of the wing out to a distance of several wing spans in the y-direction, or due to a sheet of doublets covering the wing surface. The latter is recommended for economy reasons.

The computer program in this report may be used to refine the doublet box method of Rodemich (3) in such a way as to include the (possibly very important) influence of wing thickness distribution on transonic airloads. A doublet box method similar to the one Rodemich developed (Reference 3) is recommended. The procedure is heuristically described in Appendix III. For each of a selected set of points in a sending box, the distribution of velocity potentials along ray lines throughout the zone of influence can be determined. An interpolation scheme will yield from these the velocity potentials at box centers and a numerical integration procedure will yield a velocity potential influence coefficient for each of the box centers. It will be necessary to solve a set of simultaneous equations to establish the strengths of doublets required to satisfy the tangential flow condition in the subsonic flow region. The order of the set will be equal to the number of box centers in the subsonic region on the wing. In the supersonic region the doublet strengths can be established sequentially. The use of doublets to solve unsteady supersonic flow problems has been outlined by Ashley in Reference 7.

It is recommended that this method be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict, with reasonable accuracy, the flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in available technology.

REFERENCES

1. Landahl, M. Uniformly Valid Approximate Solution for an Oscillating Source in a Transonic Flow Field. Unpublished paper, (1965).

2. Landahl, M. Approximate Solution for an Oscillating Source in a Non Uniform Transonic Stream. NAA, SID 63-1194 (August 1963)

3. Rodemich, E. R. and L. V. Andrew Unsteady Aerodynamics for Advanced Configurations, Part II - A Transonic Box Method for Planar, Lifting Surfaces. FDL-TDR-64-152, (May 1965)

4. Spreiter, J. R., and A. Y. Alksne, Thin Airfoil Theory Based on Approximate Solution of the Transonic Flow Equation, NACA Report 1359.

5. Rubbert, Paul E., Analysis of Transonic Flow by Means of Parametric Differentiation AFOSR 64-1932, MIT Fluid Dynamics Research Laboratory Report No. 65-2 (1965).

6. Wind Tunnel Tests of Four Reflection Plane Mounted .024 Scale Models Simulating The YB-70 Wing to Investigate the Effect of Camber on the Chordwise Pressure Distribution at Mach Numbers from 0.4 to 3.0. NA-61-55, TWT-50, (Unpublished)

7. Ashley, Holt, Further Studies on Aerodynamic Influence Coefficients For Supersonic Wings - Use of Doublet Sheets. NAA, SID 66-373 (July 1964).

8. Andrew, L. V. and Stenton, T. E., Unsteady Aerodynamic Forces on a Thin Wing Oscillating in Transonic Flow. AIAA Paper No. 67-16, January 23-26, 1967.

APPENDIX I. Program Listings

```
$IBFTC MAIN     SDD
C     FORTRAN PROGRAM TO COMPUTE (AND PLOT) THE PATHS OF ACOUSTIC SIG -
C     NALS (AND TRANSMISSION TIMES) ON AN AIRFOIL IN A SONIC FLOW FIELD,
C     ACCOUNTING FOR VARIATION IN LOCAL MACH NUMBER.
C     CM = COEFFICIENTS OF MACH EQUATION. (SEE SUBROUTINE FMACH )
C     PLX AND PLY ARE CONSTANTS DESCRIBING THE PLANFORM GEOMETRY.
C     THE PROGRAM ALLOWS FOR EITHER X OR Y TO BE THE INDEPENDENT VARIA-
C     BLE, DEPENDING ON THE CURRENT VALUE OF X-PRIME, WHICH SETS IVAR.
C            IF IVAR = 1,                       IF IVAR = 2,
C      YY = CURRENT VALUE OF X            YY = CURRENT VALUE OF Y
C      DYY= CURRENT VALUE OF DX           DYY= CURRENT VALUE OF DY
C      XX(1)= CURRENT VALUE OF Y-PRIME    XX(1)= CURRENT VALUE OF X-PRIME
C      XX(2) = CURRENT VALUE OF Y         XX(2) = CURRENT VALUE OF X
C      XX(3) = CURRENT VALUE OF TIME      XX(3) = CURRENT VALUE OF TIME
C      XX(4) = CURRENT VALUE OF R-BAR     XX(4) = CURRENT VALUE OF R-BAR
C      DXX(1)= Y-DOUBLE PRIME             DXX(1)= X-DOUBLE PRIME
C      DXX(2)= CURR. VALUE OF Y-PRIME     DXX(2)= CURR. VALUE OF X-PRIME
C      DXX(3)= CURR. VALUE OF DT/DX       DXX(3)= CURR. VALUE OF DT/DY
C      DXX(4)= CURRENT VALUE OF DR/DX     DXX(4)= CURRENT VALUE OF DR/DY
C
C      IVAR IS ORIGINALLY SET IN MAIN  PROGRAM, AND THEN RESET ON EACH
C     PASS THROUGH SUBROUTINE CNTRL.
C
C     WORK = WORKING AREA FOR SUBROUTINE RKS3 .
C     IFVD = FALSE AND IBKP= TRUE FOR VARIABLE INTERVAL.
C     IFVD = TRUE FOR FIXED INTERVAL.
C
C      SX = VECTOR CONTAINING COMPUTED  X- VALUES.
C      SXP = VECTOR CONTAINING COMPUTED X-PRIME VALUES.
C      SY CONTAINS COMPUTED Y VALUES
C      SYP CONTAINS COMPUTED R-BAR VALUES
C      TIM CONTAINS TRANSMISSION TIMES.
C      FM = CURRENT MACH NUMBER
C      ISORS = -1 DEFINES A SUPERSONIC SOURCE, RECEDING PATH.
C      ISROS = 0 DEFINES A SUPERSONIC SOURCE, ADVANCING PATH.
C      ISORS = 1 DEFINES A SUBSONIC SOURCE.
C      IBR = 1 FOR RIGHT BRANCH, 2 FOR LEFT
C      NCNT IS THE COUNTER FOR THE VECTORS SX,SY,SXP,SYP,TIM.  WHEN NCNT
C      = NMAX, INTEGRATION STOPS, AND THE FLOW PASSES TO NEXT PATH
C      ITRAP =1 INDICATES SIGNAL IS TRAPPED ON THE LOCAL MACH CONE.
C      DZ IS INITIAL VALUE OF INCREMENT.
C      CINF = REMOTE SPEED OF SOUND IN ROOT CHORDS PER SECOND.
C      FMINF= REMOTE MACH NUMBER
C      POTE - THE POTE MATRIX CONTAINS THE VELOCITY POTENTIALS ALONG A
C             RAY PATH, NORMALIZED ON BD .
C      FREQ =ASSUMED FREQUENCIES IN RADIANS PER SECOND.
C
      EXTERNAL DERIV, CNTRL
      COMMON
     */WORK/ WORK(50)
```

```
     */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)          SNIC0255
     */XDX/ XX(4),DXX(4),YY,DYY,DZ                                     SNIC0260
     */CM/ CM(6)                                                       SNIC0265
     */TABLE/ ATABL(4),RTABL(4)                                        SNIC0270
     */PL/  PLX(8),PLY(8)                                              SNIC0275
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX                            SNIC0280
     */SOURCE/ XO(20),YO(20)                                           SNIC0285
     */EPS/ E1,E2,FM,YMAX                                              SNIC0290
     */NNN/ NSS,NLCS,NLLS                                              SNIC0295
     */ECM/ ECM                                                        SNIC0300
     */C4/ CM2(7)                                                      SNIC0305
C                                                                      SNIC0310
 1000 FORMAT(2L12 )                                                    SNIC0315
 1010 FORMAT(6E12.8)                                                   SNIC0320
 1020 FORMAT(6I12 )                                                    SNIC0325
    3 READ (5,1020) NSORCE,NLA,NPL,NMAX,NF                             SNIC0330
      READ (5,1000)  FVD,IBKP                                          SNIC0335
      READ (5,1010) (XO(I),YO(I),I=1,NSORCE)                           SNIC0340
      READ (5,1010)  (CM(I),I=1,6 )                                    SNIC0345
      READ (5,1010) DZ,E1,E2,YMAX                                      SNIC0350
      READ(5,1010)(ATABL(I),I=1,4),(RTABL(I),I=1,4 )                   SNIC0355
      READ (5,1010) (PLX(I),PLY(I),I=1,NPL )                           SNIC0360
      READ (5,1010) CINF,FMINF,TAU,TSAA                                SNIC0365
C                                                                      SNIC0370
C     TAU=MAX. (T/C), TSAA = TANGENT OF SEMI-APEX ANGLE                SNIC0375
      DIMENSION FREQ(10), POTE(101,2,10 )                              SNIC0380
      READ (5,1010) (FREQ(I),I=1,NF )                                  SNIC0385
C                                                                      SNIC0390
      DIMENSION  XSO(40),YSO(40)                                       SNIC0395
      ECM = CINF*SQRT(5.0+FMINF**2)                                    SNIC0400
      ECM=1.0/ECM                                                      SNIC0405
      CALL SONK(40,NXY,YMAX,YSO,XSO,IER )                              SNIC0410
 2000 FORMAT(49H0 ERROR IN SUBROUTINE SONIC. CHECK MACH CONSTANTS )    SNIC0415
      GO TO (1,2), IER                                                 SNIC0420
    2 WRITE (6,2000)                                                   SNIC0425
    1 CONTINUE                                                         SNIC0430
      NVAR =4                                                          SNIC0435
C     NVAR IS THE NUMBER OF VARIABLES                                  SNIC0440
      CM2(1) =0.3                                                      SNIC0445
      CM2(2) =0.7                                                      SNIC0450
      CM2(3) =ATAN(1./TSAA)                                            SNIC0455
      CM2(4) =TAU                                                      SNIC0460
      CM2(5) =1.18*TSAA                                                SNIC0465
      CM2(6) =.04                                                      SNIC0470
      CM2(7)=FMINF                                                     SNIC0472
C     DEVELOP LAMDAS                                                   SNIC0475
      NL=2*(NLA/2)                                                     SNIC0480
C     THERE WILL ACTUALLY BE NL VALUES. IF NLA IS EVEN,NL=NLA. BUT NL= SNIC0485
C     NLA - 1  IF NLA IS ODD.                                          SNIC0490
      NL1=NL-1                                                         SNIC0495
```

```
      NL2 =NL/2                                                         SNIC0500
      XN= NL2*(NL2+1)                                                   SNIC0505
      DG= 6.28318/XN                                                    SNIC0510
      AL(1)=0.                                                          SNIC0515
      DO 10 J=3,NL1,2                                                   SNIC0520
      XJ = (J-1)/2                                                      SNIC0525
      J1 = J-1                                                          SNIC0530
      AL(J)=AL(J-2)+XJ*DG                                               SNIC0535
   10 AL(J1)=-AL(J)                                                     SNIC0540
      AL(NL)= 3.14159                                                   SNIC0545
C     SET UP GRID LIMITS                                                SNIC0550
      XU=0.                                                             SNIC0555
      XL=1.                                                             SNIC0560
      YL=-YMAX                                                          SNIC0565
      YR = YMAX                                                         SNIC0570
      CALL LIMITI(YL,YR,XL,XU)                                          SNIC0575
      DO 600 NS=1,NSORCE                                                SNIC0580
      NSS = NS                                                          SNIC0585
      CALL GRAPH(1,42,-NPL,PLY,PLX,2H Y,2H X,15H ACOUSTIC PATHS )       SNIC0590
      XOF=XO(NS)                                                        SNIC0595
      YOF=YO(NS)                                                        SNIC0600
      CALL GRAPH(0,42,-NXY,YSO,XSO )                                    SNIC0605
      NLLS = NL                                                         SNIC0610
      DO 500 NLC=1,NL                                                   SNIC0615
      NLCS = NLC                                                        SNIC0620
      ITRAP = 0                                                         SNIC0625
      CALL FMACH (XOF,YOF,FM,FMX,FMY )                                  SNIC0630
      TEST1 =FM - COS(AL(NLC))                                          SNIC0635
      TEST2 = SIN(AL(NLC))                                              SNIC0640
      IF(NLC .NE. NL ) GO TO 11                                         SNIC0645
      IF (YOF .GT. 0. ) GO TO 11                                        SNIC0650
      TEST2 = -TEST2                                                    SNIC0655
   11 IF (NLC-1) 14,12,14                                               SNIC0660
   12 IVAR=1                                                            SNIC0665
      GO TO 30                                                          SNIC0670
   14 IF(NLC-NL) 18,12,18                                               SNIC0675
   18 IF(TEST1) 22,20,22                                                SNIC0680
   20 IVAR=2                                                            SNIC0685
      GO TO 30                                                          SNIC0690
   22 TEST = TEST1/TEST2                                                SNIC0695
      ART = ABS(TEST)                                                   SNIC0700
      IF(ART-1.0) 20,12,12                                              SNIC0705
   30 CONTINUE                                                          SNIC0710
C     SET IBR                                                           SNIC0715
      FL=AL(NLC)                                                        SNIC0720
      IF(NLC-1) 32,31,32                                                SNIC0725
   31 IF(YOF) 41,41,42                                                  SNIC0730
   32 IF(NLC-NL) 36,34,36                                               SNIC0735
   34 IF(YOF) 42,41,41                                                  SNIC0740
   36 IF(FL) 42,42,41                                                   SNIC0745
```

```
   41 IBR=1
      GO TO 50
   42 IBR=2
   50 CONTINUE
C     SET ISORS
      CSL =COS(FL)
      RM=1.0/FM
      IF(FM-1.0) 60,51,51
   51 IF((FM-1.0)-E2) 52,52,58
   52 GO TO (53,54),IVAR
   53 YPR =TEST2/TEST1
      TST= 1.0-YPR**2*(FM**2 - 1.0)
   55 IF(TST-E1) 500,500,58
   54 XPR = TEST1/TEST2
      TST= XPR**2-(FM**2-1.0)
      GO TO 55
   58 IF(CSL-RM) 68,68,64
   60 ISORS=1
      GO TO 70
   64 ISORS= -1
      GO TO 70
   68 ISORS = 0
C
   70 NCNT=1
      GO TO (80,90),IVAR
C     IF IVAR=1,X IS THE INDEPENDENT VARIABLE.
C
   80 YY = XOF
      IF(TEST1) 81,81,82
   81 DYY=-DZ
      GO TO 83
   82 DYY = DZ
   83 XX(1) = TEST2/TEST1
      XX(2)=YOF
      XX(3) = 0.
      XX(4) = 0.
      GO TO 100
C     IF IVAR=2,Y IS THE INDEPENDENT VARIABLE.
   90 YY = YOF
      GO TO (91,92),IBR
   91 DYY = DZ
      GO TO 93
   92 DYY = -DZ
   93 XX(1) = TEST1/TEST2
      XX(2)= XOF
      XX(3) = 0.
      XX(4) = 0.
  100 CALL RKS3 (DERIV,CNTRL,XX,DXX,ATABL,RTABL,WORK,YY,DYY,NVAR,IFVD,IB
     1KP,NTRY,IERR )
 1070 FORMAT(1H1,30X,43H IVAR    NCNT    ISORS    IBR    ITRAP    NLCS =  )
```

```
 1080 FORMAT(1H0,27X, 6I7 )
      WRITE (6,1070)
      WRITE (6,1080) IVAR,NCNT,ISORS,IBR,ITRAP,NLCS
C
 1060 FORMAT(22H ERROR IN RKS3, IERR = I4  )
      IF(IERR) 103,140,103
  103 WRITE(6,1060) IERR
      GO TO 500
 1050 FORMAT(1H-,42X,4HXO = E16.8/ 43X,4HYO = E16.8/ 43X,10HMACH NO. = E
     116.8// 29X,31H ACOUSTIC RAY PATH FOR LAMBDA = E16.8///17X,1HX,17X,
     11HY,14X,7HX-PRIME,11X,7HR-BAR  ,12X,4HTIME// )
 1040 FORMAT(1H 7X,5E18.8)
  140 WRITE(6,1050) XO(NS),YO(NS),FM,FL
      WRITE(6,1040) (SX(I),SY(I),SXP(I),SYP(I),TIM(I),I=1,NCNT )
C
      CALL GRAPH (0,NLC,-NCNT,SY,SX )
      CALL POT (NF,FREQ,POTE )
C
 1100 FORMAT(1H1,25X,54H VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOUR
     1CE AT  )
 1110 FORMAT(1H-,42X,4HXO = E16.8/43X, 4HYO = E16.8/ 43X, 8HLAMBDA = E16
     1.8 //39X,30HALTERNATING REAL AND IMAGINARY  )
 1120 FORMAT (1H-,6X,7HOMEGA =E16.8// )
 1090 FORMAT(1H 6X,6E16.6)
C
      DO 300 N=1,NF
      IF(N .NE. 1 ) GO TO 200
      WRITE (6,1100)
      WRITE (6,1110) XO(NS),YO(NS),FL
  200 WRITE (6,1120 ) FREQ(N)
      WRITE(6,1090) (( POTE(I,K,N),K=1,2 ),I=1,NCNT )
  300 CONTINUE
  500 CONTINUE
  600 CONTINUE
      GO TO 3
      END
```

```
$IBFTC DERI     SDD
       SUBROUTINE DERIV
       COMMON
      */XDX/ XX(4),DXX(4),YY,DYY,DZ
      */CM/ CM(6)
      */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX
      */EPS/ E1,E2,FM,YMAX
      */NNN/ NSS,NLCS,NLLS
      */ECM/ ECM
C
       GO TO(10,50),IVAR
C      X IS THE INDEPENDENT VARIABLE
   10 CALL FMACH (YY,XX(2),FM,FMX,FMY )
       R=XX(1)
       DXX(2)=R
       B =FM*FM -1.0
       TSI = 1.0-R*R*B
       A =FM*FM + 5.0
       SA = SQRT(A)
       IF(B) 103,103,101
  101 BETA = SQRT(B)
       IF(ITRAP .EQ. 1) GO TO 104
       IF(ISORS .EQ. 1) GO TO 103
       IF(TSI .GT. E1) GO TO 103
       ITRAP = 1
       GO TO  104
  103 IF(TSI .GE. 0. ) GO TO 215
       ITRAP = 2
       TSI = 0.
  215 RAD = SQRT (TSI)
       DXX(4) = RAD
       RAB= 1.0/(A*B)
       TM1=FM*(FM**2 + 11.0)/B
       TM2= 2.0*FM*(FM**2+8.0)*R**2
       TM3=((RAD**3)/B)*(7.0*FM**2+5.0)
       TM4=(FM/A)*R*(6.0*R**2 +1.0)
       GO TO 105
  104 RBB = 1.0/(B**2)
       DXX(4) = 0.
  105 IF(ISORS) 11,15,18
   11 GO TO (12,13),IBR
   12 IF(ITRAP) 91,91,7
   13 IF(ITRAP) 92,92,8
   15 GO TO(16,17),IBR
   16 IF(ITRAP) 92,92,7
   17 IF(ITRAP) 91,91,8
   18 GO TO (92,91),IBR
   91 IF(R) 4,3,3
   92 IF(R) 3,3,4
C      Y IS THE INDEPENDENT VARIABLE
```

```
   50 CALL FMACH (XX(2),YY,FM,FMX,FMY )
      R = XX(1)
      DXX(2) = R
      B = FM*FM-1.0
      TSI = R*R-B
C
      A= 5.0+FM*FM
      SA = SQRT(A)
      IF(B .LT. 0. )  GO TO  108
  106 BETA = SQRT(B)
      IF(ITRAP .EQ. 1) GO TO 109
      IF(ISORS .EQ. 1) GO TO 108
       (TSI .GT. E1) GO TO 108
      ITRAP = 1
      GO TO 109
  108 IF(TSI .GE. 0.) GO TO 107
      TSI=0.
      ITRAP = 2
  107 RAD = SQRT(TSI)
      DXX(4) = RAD
      RAB = 1.0/(A*B)
      TM1=(FM/B)*(FM**2+11.0)*R**3
      TM2= 2.0*FM*(FM**2+8.0)*R
      TM3=(RAD**3/B)*(7.0*FM**2+5.0)
      TM4 =(FM/A)*(R**2+6.0)
      GO TO 110
  109 DXX(4) = 0.
  110 IF(ISORS) 52,60,68
   52 GO TO (54,56),IBR
   54 IF(ITRAP ) 1,1,5
   56 IF(ITRAP ) 2,2,6
   60 GO TO (62,64),IBR
   62 IF(ITRAP ) 2,2,5
   64 IF(ITRAP ) 1,1,6
   68 GO TO (2,1),IBR
C     FORMULAS FOR THE SECOND DERIVS FOLLOW
C
    1 IF(ABS(B) .LE. 1.E-03) GO TO 220
      DXX(1)=RAD*(-TM1 + TM2 - TM3)*FMY + TM4 *FMX
      DXX(3)=(SA*ECM/B)*(FM*XX(1)+RAD)
      GO TO 100
    2 IF(ABS(B) .GT. 1.E-03 ) GO TO 209
  220 DXX(1)=(.5/A)*(2.*R**3+R+9./R)*FMY + (FM/A)*(R**2+6.)*FMX
      DXX(3)=(1.22475*ECM)*(R+(1./R))
      GO TO 100
  209 DXX(1)= RAD*(-TM1+TM2+TM3)*FMY+TM4 * FMX
      DXX(3)=(SA*ECM/B)*(FM*XX(1)-RAD)
      GO TO 100
    3 IF(NLCS .EQ. NLLS) GO TO 4
      DXX(1)=RAB*(TM1-TM2+TM3)*FMY -TM4* FMX
```

```
      DXX(3)=(SA*ECM/B)*(FM + RAD)                                       SNIC1680
      GO TO 100                                                          SNIC1685
    4 IF(ABS(B) .GT. 1.E-03 ) GO TO 205                                  SNIC1690
  204 DXX(1)=-(.5/A)*(9.*R**4+R**2+2.)*FMY - (R/A)*(6.*R**2+1.)*FMX      SNIC1695
      DXX(3)=(1.22475*ECM)*(1.+R*R)                                      SNIC1700
      GO TO 100                                                          SNIC1705
  205 DXX(1)= RAD*(TM1-TM2-TM3)*FMY-TM4* FMX                             SNIC1710
      DXX(3)=(SA*ECM/B)*(FM - RAD)                                       SNIC1715
      GO TO 100                                                          SNIC1720
    5 DXX(1)=FM*((FMY/BETA) +FMX)                                        SNIC1725
      DXX(3)=(SA*ECM/B)* FM * XX(1)                                      SNIC1730
      GO TO 100                                                          SNIC1735
    6 DXX(1)=FM*((-FMY/BETA) +FMX )                                      SNIC1740
      DXX(3)= (SA*ECM/B)*FM*XX(1)                                        SNIC1745
      GO TO 100                                                          SNIC1750
    7 DXX(1)= -(FM*RBB)*(FMY+BETA*FMX )                                  SNIC1755
      DXX(3)= (SA*ECM/B)*FM                                              SNIC1760
      GO TO 100                                                          SNIC1765
    8 DXX(1)=FM*RBB*(-FMY+BETA*FMX)                                      SNIC1770
      DXX(3)= (SA*ECM/B)*FM                                              SNIC1775
  100 IF(DYY .LT. 0. )  GO TO 31                                         SNIC1780
      DXX(3)  = ABS(DXX(3))                                              SNIC1785
      GO TO 32                                                           SNIC1790
   31 DXX(3)  = - 1.0*(ABS(DXX(3)))                                      SNIC1795
      DXX(4)  = -1.0*(ABS(DXX(4)))                                       SNIC1800
   32 RETURN                                                             SNIC1805
      END                                                                SNIC1810
```

```
$IBFTC CONT     SDD
       SUBROUTINE CNTRL(NTRY)
       COMMON
      */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)
      */XDX/ XX(4),DXX(4),YY,DYY,DZ
      */CM/ CM(6)
      */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX
      */EPS/ E1,E2,[illegible]M,YMAX
      */NNN/ NSS,NLCS,NLLS
       IF(NCNT .NE. 1 ) GO TO 6
       NCO = 1
       IF (NR .EQ. 1) GO TO 6
       NR = 1
       IF(ABS(DXX(1)*DYY ) .LE. .25) GO TO 6
     4 DYY = .5*DYY
       IF(ABS(DXX(1)*DYY ) .LE. .25) GO TO 7
       GO TO 4
     7 NTRY = 4
       RETURN
     6 IF(ABS(XX(1)).LT. 1.0 ) GO TO 20
     1 NTRY =4
       GO TO (2,3),IVAR
     2 IVAR=2
       GO TO 5
     3 IVAR=1
C       SWITCH VARIABLES,SET NEW INITIAL CONDITIONS
     5 SAV =YY
       DYY = DYY*XX(1)
    10 YY = XX(2)
       XX(1)=1.0/XX(1)
       XX(2)=SAV
       RETURN
    20 GO TO (25,35),IVAR
C      STORE CURRENT VALUES WHERE X IS INDEPENDENT VARIABLE.
    25 SX(NCNT) = YY
C      CHANGE IBR WHEN Y-PRIM PASSES THROUGH ZERO
       IF(ABS(XX(1)) .GT. 1.0 E-02) GO TO 15
       IF((DXX(1)*DYY*XX(1)) .GE. 0.0) GO TO 15
       IF(NCO .EQ. 2) GO TO 15
       NCO = 2
       XX(1)=-XX(1)
       NTRY = 4
       GO TO (11,12), IBR
    11 IBR =2
       GO TO 19
    12 IBR = 1
       GO TO 19
    15 IF(NCO .NE. 2 ) GO TO 19
       IF(ABS(XX(1)) .LT. 1.0 E-01) GO TO 19
       NCO = 1
```

```
   19 IF(XX(1) .NE. 0.0 ) GO TO 27
   26 SXP(NCNT)= UNDEF
      GO TO 28
   27 SXP(NCNT)= 1.0/XX(1)
   28 SY(NCNT) = XX(2)
      SYP(NCNT) =XX(4)
      TIM(NCNT) = XX(3)
      GO TO 50
   35 SX(NCNT)=XX(2)
      TIM(NCNT) = XX(3)
      SXP(NCNT)=XX(1)
      SY(NCNT)=YY
      SYP(NCNT) = XX(4)
   50 CONTINUE
C     NOW TEST FOR EXIT CONDITIONS
      IF(ITRAP .NE. 2) GO TO 51
      ITRAP = 0
      NCNT = NCNT - 1
      GO TO 100
   51 IF(ITRAP) 60,60,52
   52 TEST =FM-1.0
      IF(TEST) 100,100,53
   53 IF (TEST-E2 ) 100,100,60
   60 IF (SX(NCNT)) 100,70,70
   70 IF (SX(NCNT)-1.0) 80,100,100
   80 AY =ABS(SY(NCNT))
      IF(AY-YMAX) 105,100,100
  105 IF(NCNT-NMAX) 110,100,100
  100 NTRY = 2
      NR = 0
      RETURN
  110 NCNT = NCNT + 1
      RETURN
      END
```

```
$IBFTC MACH                                                                        SNIC2235
C      MASTER SUBR., M, MX, MY                                                      SNIC2245
      SUBROUTINE FMACH(FX,FY,FMS,FMXS,FMYS)                                         SNIC2240
      COMMON                                                                        SNIC2250
     */C4/ CM2(7)                                                                   SNIC2255
      EQUIVALENCE (A,CM2(1)),(B,CM2(2)), (AL,CM2(3)), (TAU,CM2(4)),(AK,             SNIC2260
     * CM2(5)), (R1,CM2(6)) ,(FMINF,CM2(7))                                         SNIC2265
      AY=ABS(FY)                                                                    SNIC2275
      AYY = ABS(AK*FX)                                                              SNIC2280
      IF(AY .LE. AYY) GO TO 200                                                     SNIC2285
      SK = 1./ (SQRT(1.+AK*AK))                                                     SNIC2290
      T =(AY-AYY)*SK                                                                SNIC2295
  100 CALL FMAC1 (FX,AYY,FMS,FMXS,FMYS)                                             SNIC2300
      CALL FMAC2 (FX,AY,A,B,AL,TAU,D1FM,D1MX,D1MY )                                 SNIC2305
      CALL FMAC2 (FX,AYY,A,B,AL,TAU,D2FM,D2MX,D2MY)                                 SNIC2310
C                                                                                   SNIC2315
      FMS = FMS -0.6*FMINF*(D1FM-D2FM)                                              SNIC2320
      FMXS= FMXS+FMYS*AK-0.6*FMINF*(D1MX-D2MX-AK*D2MY)                              SNIC2325
      FMYS = -0.6*FMINF*D1MY*(AY/FY)                                                SNIC2330
      IF (T .GE. R1) GO TO 300                                                      SNIC2335
  120 CALL FMAC1 (FX,FY,SM,SMX,SMY )                                                SNIC2340
      ARG =1.57079*T/R1                                                             SNIC2345
      SI = SIN(ARG)                                                                 SNIC2350
      SMO = SI*SI                                                                   SNIC2355
      FMS=(FMS-SM)*SMO + SM                                                         SNIC2360
      FMXS=(FMXS-SMX)*SMO + SMX                                                     SNIC2365
      FMYS=(FMYS-SMY)*SMO +SMY                                                      SNIC2370
      GO TO 300                                                                     SNIC2375
  200 CALL FMAC1(FX,FY,FMS,FMXS,FMYS)                                               SNIC2380
  300 CONTINUE                                                                      SNIC2385
      RETURN                                                                        SNIC2390
      END                                                                           SNIC2395
```

```
$IBFTC MAC2     SDD
      SUBROUTINE FMAC2(X,Y,A,B,AL,TAU,DELCP,DDXCP,DDYCP)
C
C     SUBROUTINE COMPUTES DELTA CP
C
      CS= COS(AL)
      CS1=1./(SQRT(1.+((1.-A)**2)*(CS**2)))
      CS2=1./(SQRT(1.+((1.-B)**2)*(CS**2)))
      TA = SIN(AL)/CS
      TA1=(1.-A)*TA
      TA2=(1.-B)*TA
      EPS=TAU/(2.*3.1415927*A*CS)
      EPS1= EPS*CS/CS1
      EPS2 = EPS*A*CS/((1.-B)*CS2)
      EDS = 1.0 - EPS
      EDS1 = EPS1 + 1.0
      EDS2 = EPS2 + 1.0
      S = ABS(X/TA)
      S1=(X-A)/TA1
      S2=(X-B)/TA2
      Q1=ABS(Y-S)
      Q2=ABS(Y+S)
      Q3=ABS(Y-S1)
      Q4=ABS(Y+S1)
      Q5 =ABS(Y-S2)
      Q6 =ABS(Y+S2)
      FAC =2.*CS/TA
      FAC1=2.*CS1/TA1
      FAC2=2.*CS2/TA2
      DEL =-FAC*(Q1**EPS+Q2**EPS-2.*S**EPS)
      DDX=-FAC*(-1./(Q1**EDS)+1./(Q2**EDS)-2./(S**EDS)) * EPS /TA
      DDY =-FAC*(1./(Q1**EDS)+1./(Q2**EDS))             * EPS
      IF (S1) 10,10,5
   10 DELCP= DEL
      DDXCP= DDX
      DDYCP= DDY
      GO TO 50
    5 DEL1=-FAC1*(1./(Q3**EPS1)+1./(Q4**EPS1)-2./(S1**EPS1))
      DDX1=FAC1*(-1./(Q3**EDS1)+1./(Q4**EDS1)-2./(S1**EDS1)) * EPS1 /TA1
      DDY1= FAC1*(1./(Q3**EDS1)+1./(Q4**EDS1))               * EPS1
      IF (S2) 20,20,30
   20 DELCP= DEL+ DEL1
      DDXCP= DDX+ DDX1
      DDYCP= DDY+ DDY1
      GO TO 50
   30 DEL2=-FAC2*(1./(Q5**EPS2)+1./(Q6**EPS2)-2./(S2**EPS2))
      DDX2=FAC2*(-1./(Q5**EDS2)+1./(Q6**EDS2)-2./(S1**EDS2)) * EPS2 /TA2
      DDY2= FAC2*(1./(Q5**EDS2)+1./(Q6**EDS2))               * EPS2
      DELCP = DEL+ DEL1 + DEL2
      DDXCP= DDX+ DDX1 + DDX2
```

```
      DDYCP= DDY+ DDY1 + DDY2
   50 RETURN
      END
```

```
$IBFTC MAC1     SDD                                                       SNIC2665
      SUBROUTINE FMAC1(FX,FY,FMS,FMXS,FMYS)                               SNIC2670
C                                                                         SNIC2675
C     SUBROUTINE COMPUTES MACH NO, MX, MY.                                SNIC2680
C          FX = X                              FY = Y                     SNIC2685
C          FMS = MACH NO.                      FMXS= PARTIAL M W/RESP TO X SNIC2690
C          FMYS= PARTIAL M W/RESP TO Y                                    SNIC2695
C      EQ. FOR MACH IS M=CM(2)+EXP(-CM(1)*Y**2/X)*(CM(3)*X+CM(4)*X**2+    SNIC2700
C      CM(5)*Y**2+CM(6)*Y**4 )                                            SNIC2705
      COMMON                                                              SNIC2710
     */CM/ CM(6)                                                          SNIC2715
C                                                                         SNIC2720
      EQUIVALENCE                                                         SNIC2725
     1      ( C ,CM(1)),    ( FMO,CM(2)),      (A1 ,CM(3)),    ( A2 ,CM(4)),SNIC2730
     2      (A3 ,CM(5)),    ( A4 ,CM(6))                                  SNIC2735
      IF(FX .EQ. 0.) GO TO 5                                              SNIC2740
      ARG1 =(-C*FY**2)/FX                                                 SNIC2745
      ARG1 = - ABS(ARG1)                                                  SNIC2750
      IF (ABS(ARG1) .GE. 50.) GO TO 5                                     SNIC2755
      ARG2 = A1*FX+A2*FX**2 +A3*FY**2 +A4*FY**4                           SNIC2760
C                                                                         SNIC2765
      ARG3 = A1+ 2. * A2*FX                                               SNIC2770
      ARG4 = 2.*A3*FY +4.*A4*FY**3                                        SNIC2775
      EX = EXP(ARG1)                                                      SNIC2780
      GO TO 10                                                            SNIC2785
    5 FMS = FMO                                                           SNIC2790
      FMXS = 0.                                                           SNIC2795
      FMYS = 0.                                                           SNIC2800
      RETURN                                                              SNIC2805
   10 FMS = FMO +EX* ARG2                                                 SNIC2810
      FMXS= EX*((-ARG1/FX)* ARG2 +ARG3)                                   SNIC2815
      PAUL= -2.*C*FY/FX                                                   SNIC2820
      FMYS= EX*( PAUL*ARG2 + ARG4)                                        SNIC2825
      RETURN                                                              SNIC2830
      END                                                                 SNIC2835
```

```
$IBFTC SONI     SDD
      SUBROUTINE SONK(NM,NCR,YM,FY,FX,IER  )
C
C     NM = MAX NO OF X,Y ALLOWED. MUST EQUAL DIMENSION OF X,Y, IN MAIN
C     NCR = NO OF X,Y ACTUALLY COMPUTED
C     YM = MAX. ALLOWABLE VALUE OF Y
C     FX = X-VALUES
C     FY = Y-VALUES
C     IER = 1 IS NORMAL RETURN
C     IER = 2 INDICATES AN ERROR
C     CM= MACH CONSTANTS IN THE EQUATION  M=EXP(-CM(1)*Y**2/X)*(CM(3)*X
C     +CM(4)*X*X+CM(5)*Y*Y+CM(6)*Y**4)  +CM(2).
C     THE SUBROUTINE COMPUTES A SET OF X AND Y VALUES ON THE WING WHERE
C     M= 1
C
C
      COMMON
     */CM/ CM(6)
      DIMENSION FX(1),FY(1)
      IER =1
      C=CM(1)
      FMO=CM(2)
      A1 =CM(3)
      A2 =CM(4)
      A3 =CM(5)
      A4 =CM(6)
      FIRST COMPUTE X WHEN Y=0
      ARG = A1**2 -4.*A2*(FMO-1.)
      IF(ARG .GE. 0.0) GO TO 2
    1 IER = 2
      RETURN
    2 FX(1) =(.5/A2)*(-A1+SQRT(ARG))
      FY(1) = 0.
      IF(FX(1) .LT. 0.0) GO TO 1
      IF(FX(1) .LT. 1.0) GO TO 4
      FX(1)=(.5/A2)*(-A1-SQRT(ARG))
      IF(FX(1) .LT. 0.0) GO TO 1
      IF(FX(1) .GE. 1.0) GO TO 1
    4 NCR = 2
   10 NC1= NCR - 1
      FX(NCR)= FX(NC1)+.01
      X= FX(NCR)
      R = C/X
      B = X*(A1+A2*X)
      TO = FY(NC1)**2
      TM1 = A3-R*B
      TM2 = 2.*A4-R*A3
      TM3 = R*A4
      TM4 = 2.*A4+R*(R*B-2.*A3)
      TM5 = R*(R*A3-4.*A4)
```

```
      TM6 = R*R*A4
      IMAX =1
   12 ET =EXP(-R*TO)
      FT= ET*(B+A3*TO+A4*TO*TO)+FMO-1.
      FPT =ET*(TM1+TM2*TO-TM3*TO**2)
      FPPT =ET*(TM4+TM5*TO+TM6*TO**2)
      HO = -FT/FPT
      IF((FT*FPPT) .GE. 0.0) GO TO 14
      HO = .75*HO
   14 TO =TO+HO
      IMAX =IMAX +1
 1000 FORMAT(52H0 COMPUTATION FOR SONIC LINE WILL NOT CONVERGE, HO = E
     116.8 )
      IF(IMAX .LT. 10 ) GO TO 18
      WRITE (6,1000) HO
      GO TO 1
   18 IF(HO .GT. .0001) GO TO 12
      FY(NCR)= SQRT(TO)
      IF(NCR .GE. NM ) GO TO 20
      IF(FY(NCR) .GE. YM) GO TO 20
      IF(FX(NCR) .GE. 1.0) GO TO 20
      NCR = NCR +1
      GO TO 10
   20 RETURN
      END
```

```
$IBFTC POTE
      SUBROUTINE POT (NFR,FR,P)
      COMMON
     */XYZ/ SX(101),SXP(101),SY(101),SYP(101),AL(41),TIM(101)
     */CM/ CM(6)
     */ICNT/ IVAR,NCNT,ISORS,IBR,ITRAP,NMAX
     */SOURCE/ XO(20),YO(20)
     */EPS/ E1,E2,FM,YMAX
     */NNN/ NSS,NLCS,NLLS
C
      DIMENSION  FR(10),P(101,2,10)
      CON=-.25/3.14159
      XS =XO(NSS)
      YS =YO(NSS)
      DO 100 N=1,NCNT
      X=SX(N)
      Y=SY(N)
      T = TIM(N)
      RBAR = SYP(N)
   10 DO 30 NF=1,NFR
      IF(RBAR) 12,14,16
   12 P(N,1,NF)=0.
      P(N,2,NF)=0.
      GO TO 30
   14 P(N,1,NF)=UNDEF
      P(N,2,NF)=UNDEF
      GO TO 30
   16 IF(RBAR .LE. 1.E-9) GO TO 14
      FACT = CON/RBAR
      ARG = FR(NF)*T
      CO = COS(ARG)
      SI = SIN(ARG)
      P(N,1,NF)= CO*FACT
      P(N,2,NF)= -SI*FACT
   30 CONTINUE
  100 CONTINUE
      RETURN
      END
```

```
$IBFTC RKS3*    RUNGE-KUTTA, FORTRAN IV, VERSION 13, SHARE D2*ATFRKS3
      SUBROUTINE RKS3 (DERIV,CNTRL,Y,DY,ATABL,RTABL,WORK,X,DX,N,IFVD
     1                ,IBKP,NTRY,IERR)
      EXTERNAL DERIV,CNTRL
      INTEGER N,NTRY,IERR
      LOGICAL IFVD,IBKP
      REAL Y,DY,ATABL,RTABL,X,DX
      DIMENSION Y(N),DY(N),ATABL(N),RTABL(N)
      DIMENSION WORK(1)
C     DIMENSION WORK(9*N+8)
      CALL RKINT (DERIV,CNTRL,Y,DY,ATABL,RTABL,WORK(1),WORK(3),WORK(5)
     1           ,WORK(7),WORK(9),WORK(2*N+9),WORK(4*N+9),WORK(6*N+9)
     2           ,WORK(7*N+9),WORK(8*N+9),X,DX,N,IFVD,IBKP,NTRY,IERR)
      RETURN
      END
$IBFTC RKINT*   CALLED BY RKS3, RUNGE-KUTTA, F 4, V13, SHARE D2*ATFRKS3
      SUBROUTINE RKINT (DERIV,CNTRL,REALY,DY,ATABL,RTABL,DELTAX,X,XHALF
     1                 ,XZERO,Y,YHALF,YZERO,DYHALF,DYZERO,DELTAY,REALX
     2                 ,DX,N,IFVD,IBKP,NTRY,IERR)
      EXTERNAL DERIV,CNTRL
      INTEGER N,NTRY,IERR
      LOGICAL IFVD,IBKP
      REAL REALY,DY,ATABL,RTABL,DELTAX,DYHALF,DYZERO,DELTAY,REALX,DX
      DOUBLE PRECISION X,XHALF,XZERO,Y,YHALF,YZERO
      DIMENSION REALY(N),DY(N),ATABL(N),RTABL(N),Y(N),YHALF(N),YZERO(N)
     1         ,DYHALF(N),DYZERO(N),DELTAY(N)
      IERR = 0
   10 DELTAX = DX
      X = REALX
      DO 20 I=1,N
   20 Y(I) = REALY(I)
      CALL DERIV
      GO TO 200
   30 IF (DX .EQ. 0.) GO TO 230
      DELTAX = DX
      DX2 = DX/2.
      DX4 = DX/4.
      XZERO = X
      DO 40 I=1,N
      YZERO(I) = Y(I)
   40 DYZERO(I) = DY(I)
      DO 110 J=1,2
      XHALF = X
      X = X+DX4
      REALX = X
      DO 50 I=1,N
      DELTAY(I) = DY(I)*DX4
      YHALF(I) = Y(I)
      Y(I) = Y(I)+DELTAY(I)
   50 REALY(I) = Y(I)
```

```
      CALL DERIV
      DO 60 I=1,N
      DELTAY(I) = DELTAY(I)+DY(I)*DX2
      Y(I) = YHALF(I)+DY(I)*DX4
   60 REALY(I) = Y(I)
      CALL DERIV
      X = XHALF+DX2
      REALX = X
      DO 70 I=1,N
      DELTAY(I) = DELTAY(I)+DY(I)*DX2
      Y(I) = YHALF(I)+DY(I)*DX2
   70 REALY(I) = Y(I)
      CALL DERIV
      DO 80 I=1,N
      DELTAY(I) = (DELTAY(I)+DY(I)*DX4)/3.
      Y(I) = YHALF(I)+DELTAY(I)
   80 REALY(I) = Y(I)
      CALL DERIV
      GO TO (90,110),J
   90 DO 100 I=1,N
  100 DYHALF(I) = DY(I)
  110 CONTINUE
      IF (IFVD) GO TO 200
      ERRMAX = 0
      DO 120 I=1,N
      ERR = ATABL(I)+ABS(RTABL(I)*REALY(I))
      IF (ERR .EQ. 0.) GO TO 220
      SR = (DYZERO(I)+4.*DYHALF(I)+DY(I))/3.*DX2
  120 ERRMAX = AMAX1(ERRMAX,ABS (SR-(REALY(I)-SNGL(YZERO(I))))/ERR)
      IF (ERRMAX-1.) 130,170,160
  130 IF (ERRMAX-.75) 140,200,170
  140 IF (ERRMAX-.075) 150,200,200
  150 DX = DX*1.5845932
      GO TO 200
  160 DX = DX/1.5848932
      IF (.NOT. IBKP) GO TO 180
      ERRMAX = ERRMAX/10.
      IF (ERRMAX .GT. 1.) GO TO 160
      GO TO 180
  170 DX = DX/1.5848932
      GO TO 200
  180 X = XZERO
      DO 190 I=1,N
      Y(I) = YZERO(I)
  190 DY(I) = DYZERO(I)
      GO TO 30
  200 NTRY = 1
      CALL CNTRL (NTRY)
      GO TO (30,210,180,10),NTRY
  210 RETURN
```

```
220 IERR = 1
    RETURN
230 IERR = -1
    RETURN
    END
```

APPENDIX II. Sample Input and Output

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. ___

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	6	73	NSOURCE (NUMBER OF SOURCE POINTS, 20 MAXIMUM)
13	2 0		NLA (NUMBER OF Λ PER SOURCE, 40 MAXIMUM)
25	5		NPL (NUMBER OF PLANFORM COORDINATES, 8 MAXIMUM)
37	7 0		NMAX (LIMIT NUMBER OF POINTS PER PLOT, 100 MAX)
49	3	73 ... 80	NF (NUMBER OF ASSUMED FREQUENCIES, 10 MAXIMUM)
61		1	
1	F A L S E		IFVD LOGICAL WORDS - VARIABLE INTERVAL MODE
13	T R U E		IBKP IF IFVD = FALSE AND IBKP = TRUE THIS CHOICE
25			IS RECOMMENDED. FIXED INTERVAL IF
37			IFVD = TRUE.
49		73 ... 80	
61		2	
1			
13			
25			
37			
49		73 ... 80	
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. __

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 6 + 00		XO(1) COORDINATES OF SOURCE POINTS.
13	0 + 00		YO(1) (ALL GEOMETRY IS NORMALIZED ON b_o, THE
25	2 2 + 00		XO(2) DISTANCE FROM MOST FORWARD TO MOST AFT
37	0 4 + 00		YO(2) PORTION OF THE WING.)
49	3 8 + 00	73 80	
61	1 4 + 00	3	
1	4 8 + 00		
13	1 8 + 00		
25	5 + 00		
37	0 + 00		
49	6 + 00	73 80	XO(NSØRCE)
61	2 + 00	4	YO(NSØRCE)
1			
13			
25			
37			
49		73 80	
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ____ of ____ JOB NO. __

	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 +01	73 80	CM(1) COEFFICIENTS IN MACH NUMBER EQUATION
13	957925 +00		CM(2) $M = CM(2) + EXP\{-CM(1)(Y^2/X)\}.$
25	126673 +00		$\{CM(3)x + CM(4)\ x^2 + CM(5)Y^2$
37	63059 +00		$+ CM(6)\ Y^{\frac{1}{2}}\}$
49	790155 +00		THE COEFFICIENTS WERE DETERMINED BY A
61	891569 +00	5	CM(6) LEAST-SQUARE PROCEDURE.
1	2 -01	73 80	DZ INITIAL VALUE OF INCREMENT
13	1 -01		E1 TEST WORD FOR TRAPPING SIGNAL ON LOCAL M.L.
25	1 -02		E2 TEST WORD FOR STOPPING TRAPPED SIGNAL ON SONIC LINE
37	46631 +00		YMAX NORMALIZED SEMI-SPAN
49			
61		6	
1		73 80	
13			
25			
37			
49			
61			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. ______ PROGRAMMER ______ DATE ______ PAGE ___ of ___ JOB NO. ___

Col.	NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1 -04		ATABL(1) ATABL AND RTABL DETERMINE THE ACCURACY
13	1 -04		ATABL(2) REQUIREMENTS FOR DECREASING OR IN-
25	1 -04		ATABL(3) CREASING THE INTERVAL IN THE VARIABLE
37	1 -04		ATABL(4) INTERVAL MODE.
49	1 -03	73 80	RTABL(1)
61	1 -03	7	RTABL(2)
1	1 -03		RTABL(3)
13	1 -03		RTABL(4)
25			
37			
49		73 80	
61		8	
1	1 +01		PLX(1) PLANFORM COORDINATES. LIST ALL CORNERS
13	-46631 +00		PLY(1) STARTING FROM LEFT ALONG LEADING EDGE
25	0 +00		PLX(2) TOWARD THE RIGHT, AND AGAIN STARTING
37	0 +00		PLY(2) AT LEFT, ALONG TRAILING EDGE TOWARD
49	1 +01	73 80	PLX(3) THE RIGHT. ALL GEOMETRY IS NORMALIZED
61	46631 +00	9	PLY(3) on b_o.
1			

FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.______ PROGRAMMER______ DATE______ PAGE____ of____ JOB NO._

Col.	NUMBER		IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	1	+ 0 1		PLX(4)
13	- 4 6 6 3 1	+ 0 0		PLY(4)
25	1	+ 0 1		PLX(5)
37	4 6 6 3 1	+ 0 0		PLY(5)
49			73 80	$
61			1 0	
1	9 4 8 5 3	+ 0 1		CINF REMOTE SPD. OF SND. IN bo UNITS/SEC.
13	1	+ 0 1		FMINF REMOTE MACH NUMBER
25	0 2	+ 0 0		TAU MAXIMUM THICKNESS RATIO (T/C)
37	4 6 6 3 1	+ 0 0		TSAH TANGENT SEMI-APEX ANGLE
49			73 80	
61			1 1	
1	1	+ 0 2		FREQ(1) FREQUENCIES, FOR COMPUTATION
13	2	+ 0 2		FREQ(2) OF POTENTIALS, (RADIANS/SEC)
25	0	+ 0 0		FREQ(NF)
37				
49			73 80	
61			1 2	

IVAR	NCNT	ISORS	IBR	ITRAP	NLCS =
2	12	1	1	0	9

XO = 0.18000000E 00
YO = 0.00000000E-38
MACH NO. = 0.97311907E 00

ACOUSTIC RAY PATH FOR LAMBDA = 0.57119817E 00

X	Y	X-PRIME	R-BAR	TIME
0.18000000E 00	0.00000000E-38	0.21141810E 00	0.00000000E-38	0.00000000E-38
0.18424259E 00	0.20000000E-01	0.21257251E 00	0.72580551E-02	0.38412277E-02
0.19096695E 00	0.51691864E-01	0.21112035E 00	0.18791161E-01	0.99172437E-02
0.20135082E 00	0.10191058E 00	0.20083363E 00	0.37255190E-01	0.19553514E-01
0.20530157E 00	0.12189922E 00	0.19445186E 00	0.44702658E-01	0.23404072E-01
0.20915295E 00	0.14188787E 00	0.19213422E 00	0.52104825E-01	0.27296191E-01
0.21302884E 00	0.16187273E 00	0.19647354E 00	0.59213898E-01	0.31281433E-01
0.21700866E 00	0.18185759E 00	0.20163166E 00	0.66174245E-01	0.35289611E-01
0.22350139E 00	0.21352546E 00	0.20816299E 00	0.77123095E-01	0.41616165E-01
0.23415963E 00	0.26370616E 00	0.21631019E 00	0.94319344E-01	0.51590675E-01
0.25178554E 00	0.34322216E 00	0.22669061E 00	0.12128799E 00	0.67289645E-01
0.28126384E 00	0.46922266E 00	0.24104052E 00	0.16353054E 00	0.91917480E-01

VELOCITY POTENTIALS ALONG A RAY PATH FOR A SOURCE AT

XO = 0.18000000E 00
YO = 0.00000000E-38
LAMBDA = 0.57119817E 00

ALTERNATING REAL AND IMAGINARY

OMEGA = 0.10000000E 02

******	******	-0.109559E 02	0.421050E 00	-0.421403E 01	0.419291E 00
-0.209531E 01	0.415009E 00	-0.173162E 01	0.412835E 00	-0.147071E 01	0.411726E 00
-0.127868E 01	0.413568E 00	-0.112844E 01	0.415620E 00	-0.943756E 00	0.417118E 00
-0.733892E 00	0.416219E 00	-0.513086E 00	0.408919E 00	-0.295125E 00	0.386914E 00

OMEGA = 0.20000000E 02

******	******	-0.109317E 02	0.841478E 00	-0.415181E 01	0.834462E 00
-0.197475E 01	0.814201E 00	-0.158867E 01	0.803160E 00	-0.130527E 01	0.792965E 00
-0.108936E 01	0.786997E 00	-0.915255E 00	0.780015E 00	-0.694583E 00	0.763032E 00
-0.433042E 00	0.724092E 00	-0.146383E 00	0.639566E 00	0.128649E 00	0.469308E 00

OMEGA = 0.00000000E-38

******	******	-0.109640E 02	0.000000E-38	-0.423484E 01	0.000000E-38
-0.213601E 01	0.000000E-38	-0.178015E 01	0.000000E-38	-0.152726E 01	0.000000E-38
-0.134390E 01	0.000000E-38	-0.120255E 01	0.000000E-38	-0.103182E 01	0.000000E-38
-0.843703E 00	0.000000E-38	-0.656104E 00	0.000000E-38	-0.486622E 00	0.000000E-38

APPENDIX III. Application to the Boundary Value Problem

A procedure that may be used to match the tangential flow condition on a wing surface is, in principle, the same as that employed by Rodemich in the box method for uniform sonic flow (Reference 3). The velocity potential at a field point (x,y,z) due to a doublet sheet in its zone of influence, is

$$\bar{\phi}(x,y,z) = \frac{\partial}{\partial z} \iint_{S+W} \Delta\bar{\phi}(\xi,\eta)\phi_o(x-\xi,y-\eta,z)d\xi d\eta \qquad (39)$$

where $\Delta\phi(\xi,\eta)$ is the velocity potential discontinuity through the doublet sheet over the region S + W (the surface and its wake), and

$$\phi_o(x-\xi,y-\eta,z) = \frac{-1}{2\pi\bar{R}} \sum_{n=1}^{N} e^{-i\omega g_n} \qquad (40)$$

where $\bar{R} = \sqrt{(x-\xi)^2 + [1-M_L^2(x,y,z)]\,[(y-\eta)^2 + z^2]}$

and where N represents the number of times the wave front passes the field point. In uniform subsonic flow N equals one, in uniform supersonic flow it equals two, and in the limiting case of uniform sonic flow it equals one. As discussed previously, in uniform sonic flow the stationary portion of the perturbation wave front is not augmented by high frequency signals that follow it; instead, the pressure discontinuity is dissipated by them.

When the local flow in a non-uniform flow field is sonic the wave front gradually becomes stationary and is dissipated. Rays of this type are shown in Figures 9, 12, and 13. In certain regions of non-uniform flow a wave front may pass field points more than twice as shown in Figures 6, 7, 9, 10, and 12. These regions may be in the region of subsonic flow or in supersonic flow. Multiple crossings normally occur on receding portions of the wave front. Ray lines on advancing portions normally pass over the trailing edge before they cross. In these regions of multiple crossings of the wave front, care must be taken to establish an accurate value of N, and of each of the corresponding g_n's, $n = 1, 2, \ldots N$. A computer program that may be used to do this is contained herein. Figures 11 and 13 show that in some regions of both subsonic and supersonic flow even the receding ray lines do not cross. All of Figures 6 through 13 show that once a ray crosses the transition region at the edge of the planform it does not return to the wing region. This characteristic is important because when a doublet solution is employed a ray trace can be ignored once it reaches an edge that is not adjacent to the wake.

The next step in the procedure is to define a grid of square boxes over the region $S + W$, and assume that $\Delta\phi(\xi,\eta)$ is constant over the area of each box. For this to be a valid assumption as many as 50 boxes along the root chord may be required. The upwash adjacent to the upper surface may be written

$$\overline{W}(x,y,0+) = \lim_{z\to 0+} \frac{\phi(x,y,z)}{z}$$

or,

$$\overline{W}(x_i,y_i,0+) = \sum_{i'j'} \overline{\Delta\phi}_{i'j'} \iint_{B_{i'j'}} \psi(x_i-\xi,y_i-\eta)\,d\xi\,d\eta \tag{41}$$

i.e., the upwash at (x_i,y_i) equals the summation (over all boxes $B_{i'j'}$ that influence it), of products of the constant velocity potential discontinuities and their downwash influence coefficients. The latter are represented by the double integral of the kernel ψ over the areas of the boxes. The limits of integration and $\overline{\Delta\phi}$ of Equation (39) are not functions of z, so from Equation (40) we get

$$\psi(x_i-\xi,y_j-\eta) = \frac{-1}{2\pi} \lim_{z\to 0+} \frac{1}{z}\frac{\partial}{\partial z} \frac{\Sigma\, e^{-i\omega g_n}}{\overline{R}} \tag{42}$$

At this point it is theorized that for non-uniform flow around a nearly planar surface the variation in signal transmission time with distance normal to the surface is approximately equal to the variation in uniform flow, i.e.,

$$\frac{\partial g_n}{\partial z} = \frac{\partial}{\partial z} \frac{M(x-\xi) \mp \overline{R}}{C(M^2-1)}$$

or, performing the differentiation

$$\frac{\partial g_n}{\partial z} = \frac{\pm z}{C\overline{R}} \tag{43}$$

where the upper sign refers to the advancing portion of the wave front and the lower sign to the receding portion. C is the speed of sound. Making use of equation (43) when taking the derivative in equation (42).

$$\psi(x_i-\xi,y_i-\eta) = \frac{-1}{2\pi} \frac{\beta^2 C \pm i\omega M\overline{R}}{C\overline{R}^3} \sum_n e^{-i\omega g_n} \tag{44}$$

The g_n's are those obtained by tracing ray paths through the non-uniform flow field.

One way in which Equation (44) may be evaluated and integrated is as follows: Say for nine values of (ξ,η) on each sending box, the values of the kernel at the center of the receiving box (x_i,y_j) are evaluated. Since the ray paths are not known in advance, each of these values must be interpolated from values in its neighborhood. It is then necessary to evaluate the integral in Equation (41) given the values of the integrand at nine points in the region of integration.

The unknowns in Equation (41) are the $\overline{\Delta\phi}_{i',j'}$'s. When the center of a receiving box (x_i,y_j) lies in the subsonic flow region it lies in the zone of influence of every other point in the subsonic region and may lie in the zone of influence of a small portion of the supersonic region (Figure 9). All velocity potentials in zones of mutual influence must be determined simultaneously. Once velocity potentials have been established that meet the tangential flow conditions on the surface and the zero pressure difference condition on the wake they may be fitted with analytical expressions that have the proper edge behavior. Using these expressions, local oscillatory pressures and generalized forces may be obtained in the way outlined in Reference 3.

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) North American Rockwell Corporation International Airport Los Angeles, California	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP N/A

3. REPORT TITLE
Unsteady Aerodynamics for Advanced Configurations - Velocity Potentials in Non-Uniform Transonic Flow Over a Thin Wing

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Final Report, 1 July 1965 to 30 August 1967

5. AUTHOR(S) (First name, middle initial, last name)
Andrew, Lowell V.
Stenton, Thomas E.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
August 1968	63	7

8a. CONTRACT OR GRANT NO. AF33(615)-2896 b. PROJECT NO. 1370 c. Task 137003 d.	9a. ORIGINATOR'S REPORT NUMBER(S) FDL-TDR-64-152, Part VII 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report) NA-66-694

10. DISTRIBUTION STATEMENT
This document has been approved for public release and sale; its distribution is Unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Air Force Flight Dynamics Laboratory Wright-Patterson AFB, Ohio 45433

13. ABSTRACT

Two methods have been outlined in detail, and one of them has been mechanized, for calculating acoustic ray paths emanating from any point in a non-uniform transonic flow field surrounding a wing. It gives the ray path, and the time, for the minimum time of travel from the acoustic source point to the field point. The resulting velocity potential is also computed.

It was necessary to establish an accurate representation of the flow characteristics in the field surrounding the wing. Some ray lines travel over the planform and into the surrounding flow field. It was established that once off the planform they do not return.

Available methods predict phase lags based on the assumption that acoustic rays travel in straight lines. The results of this study show this to be a very poor approximation at transonic speeds. Therefore, it is recommended that the method presented in this report be fully developed for the purpose of calculating generalized forces on wings in harmonic motion at transonic speeds. A computer program that would predict these phase lags with reasonable accuracy, and the corresponding flutter characteristics and unsteady aerodynamic loads on a wing responding to externally applied forces, such as gusts, would fill an important gap in the available technology.

DD FORM 1 NOV 65 1473

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Unsteady Aerodynamics, Non-Uniform Transonic Flow, Velocity Potentials, Thickness Effects						

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